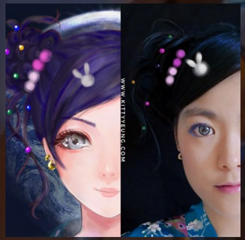
Introduction to Quantum Computing



Kitty Yeung, Ph.D. in Applied Physics

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Nov 15, 2019
Hackaday Supercon

[Microsoft Bay Area quantum computing study group]()

THEGARAGE

Resources:

[Microsoft quantum team]

Study Group Tutorial (stay tuned)

[Book: Quantum Computation and Quantum Information]

[Employee blog]

Q# documentation http://docs.microsoft.com/quantum

What is it?

Performing calculations based on the laws of quantum mechanics

1982: Feynman proposed the idea of creating machines based on the laws of quantum mechanics

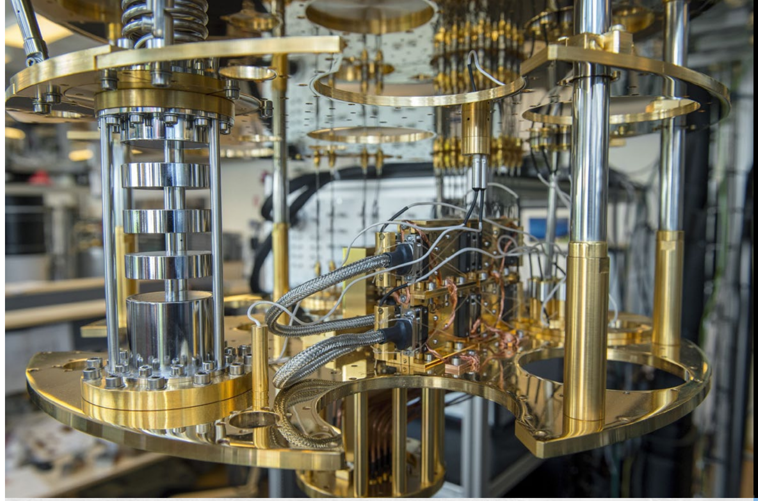
1985: David Deutsch developed Quantum Turing machine, showing that quantum circuits are universal

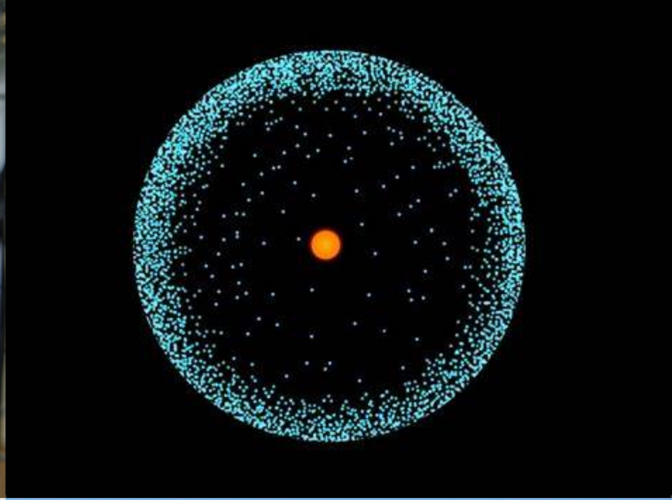
1994: Peter Shor came up with a quantum algorithm to factor very large numbers in polynomial time

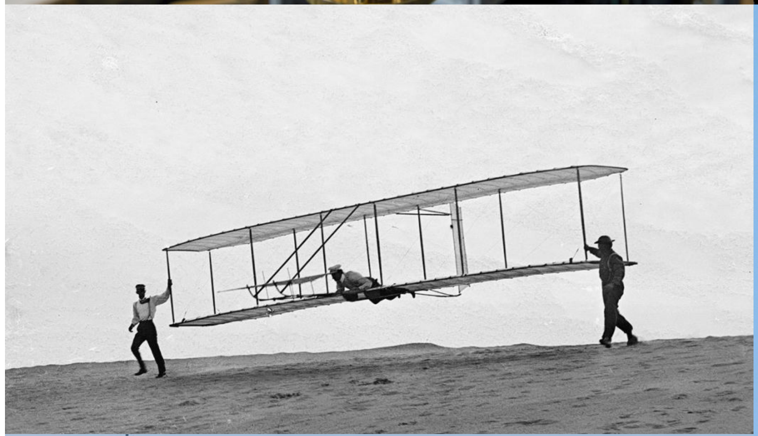
1997: Grover developed a quantum search algorithm with $O(\sqrt{N})$ complexity

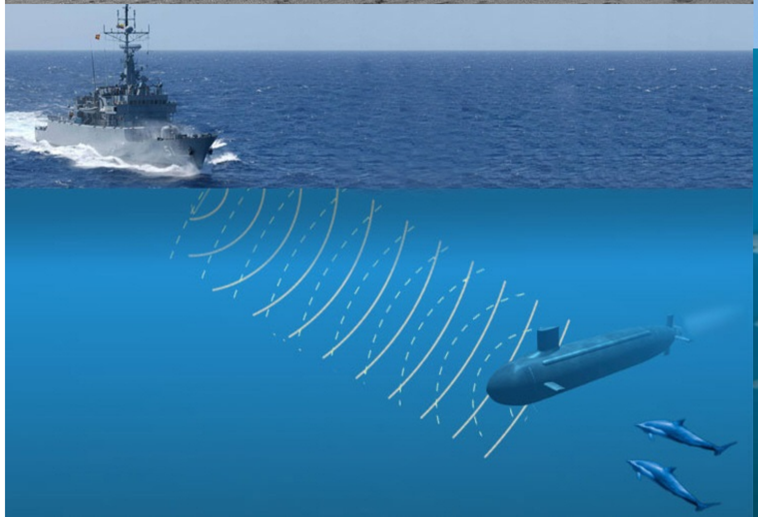
Applications

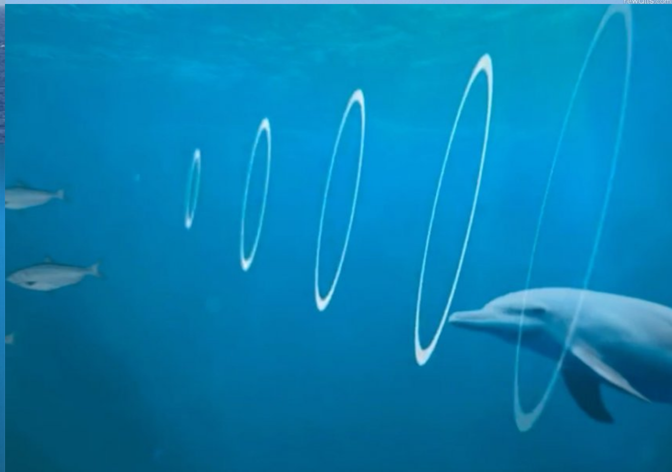
- Algorithms
- Cryptography
- Quantum simulations











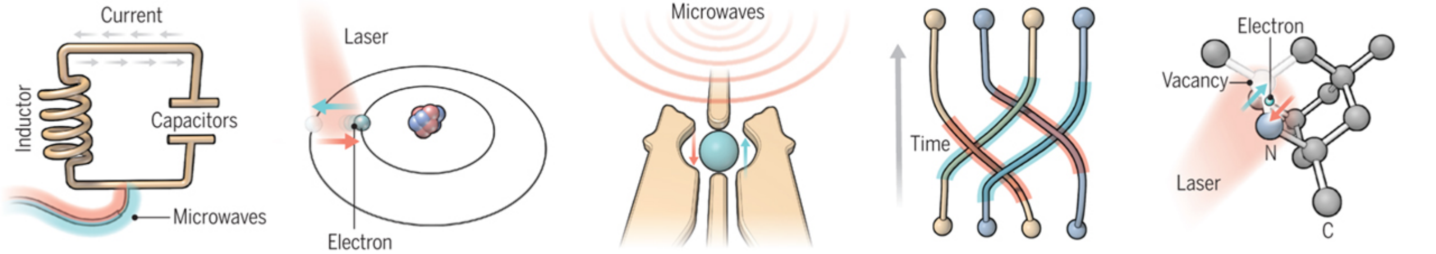
Quantum Computer Hardware

- Trapped ions
- Superconducting
- Topological

2-level system
Superposition
Entanglement
Interference

A bit of the action

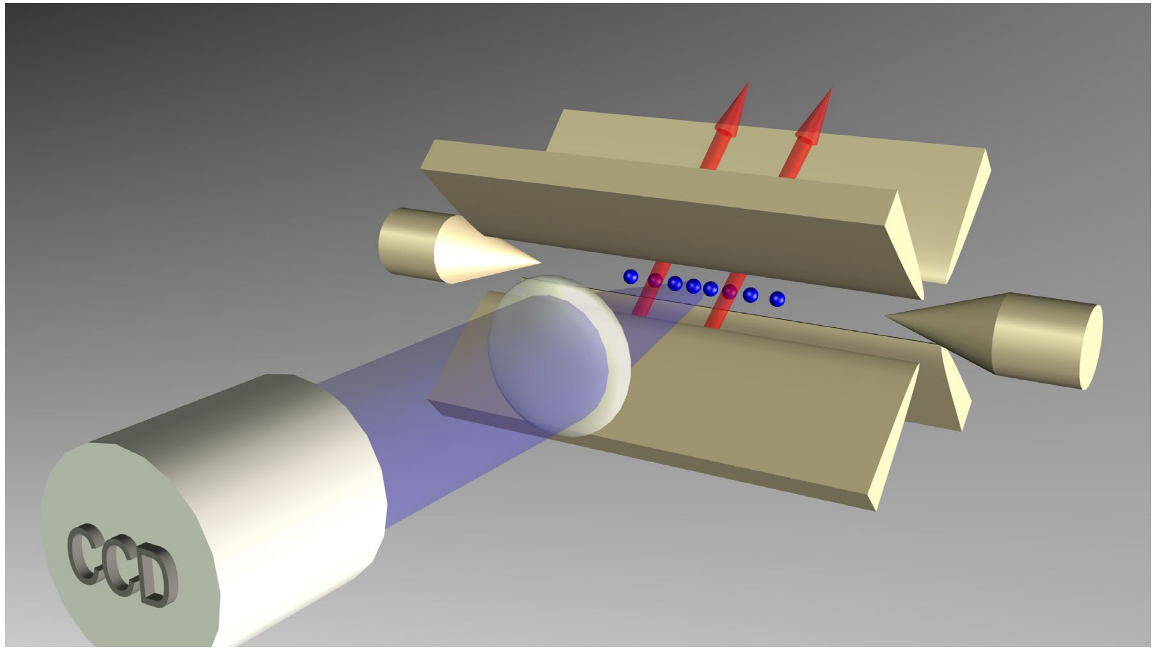
In the race to build a quantum computer, companies are pursuing many types of quantum bits, or qubits, each with its own strengths and weaknesses.



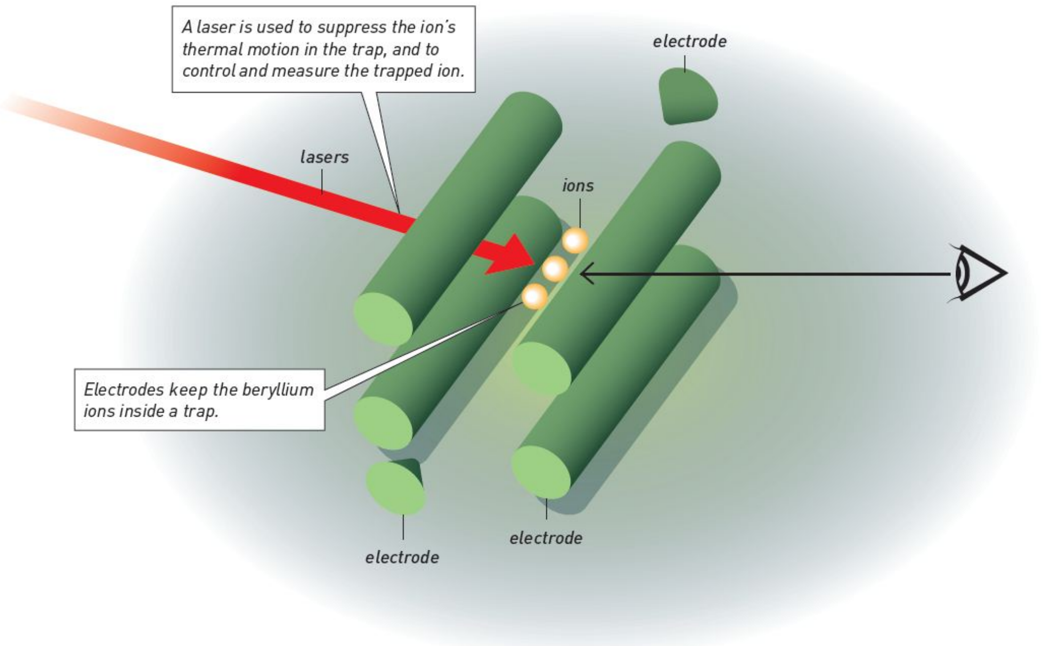
	Superconducting loops	Trapped ions	Silicon quantum dots	Topological qubits	Diamond vacancies
	A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into super-position states.	Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in superposition states.	These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.	Quasiparticles can be seen in the behavior of electrons channeled through semi-conductor structures. Their braided paths can encode quantum information.	A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.
Longevity (seconds)	0.00005	>1000	0.03	N/A	10
Logic success rate	99.4%	99.9%	~99%	N/A	99.2%
Number entangled	9	14	2	N/A	6
Company support	Google, IBM, Quantum Circuits	ionQ	Intel	Microsoft, Bell Labs	Quantum Diamond Technologies
Pros	Fast working. Build on existing semiconductor industry.	Very stable. Highest achieved gate fidelities.	Stable. Build on existing semiconductor industry.	Greatly reduce errors.	Can operate at room temperature.
Cons	Collapse easily and must be kept cold.	Slow operation. Many lasers are needed.	Only a few entangled. Must be kept cold.	Existence not yet confirmed.	Difficult to entangle.

Note: Longevity is the record coherence time for a single qubit superposition state, logic success rate is the highest reported gate fidelity for logic operations on two qubits, and number entangled is the maximum number of qubits entangled and capable of performing two-qubit operations.

Science, Dec 2016, Vol 354, Issue 6316

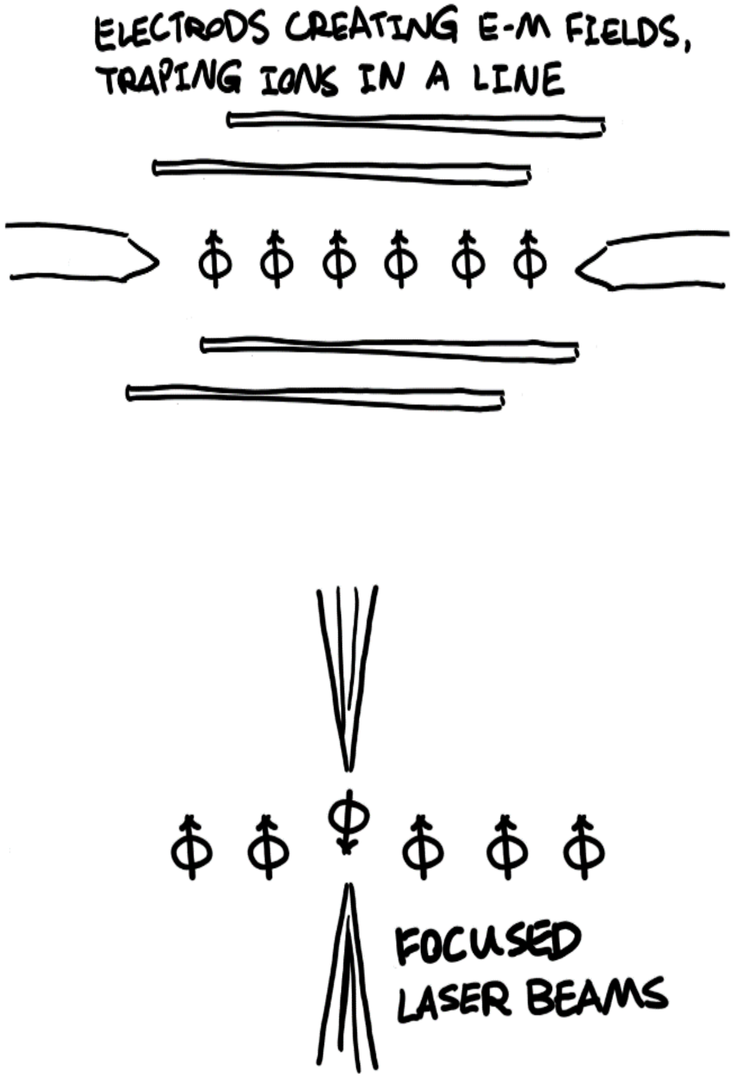


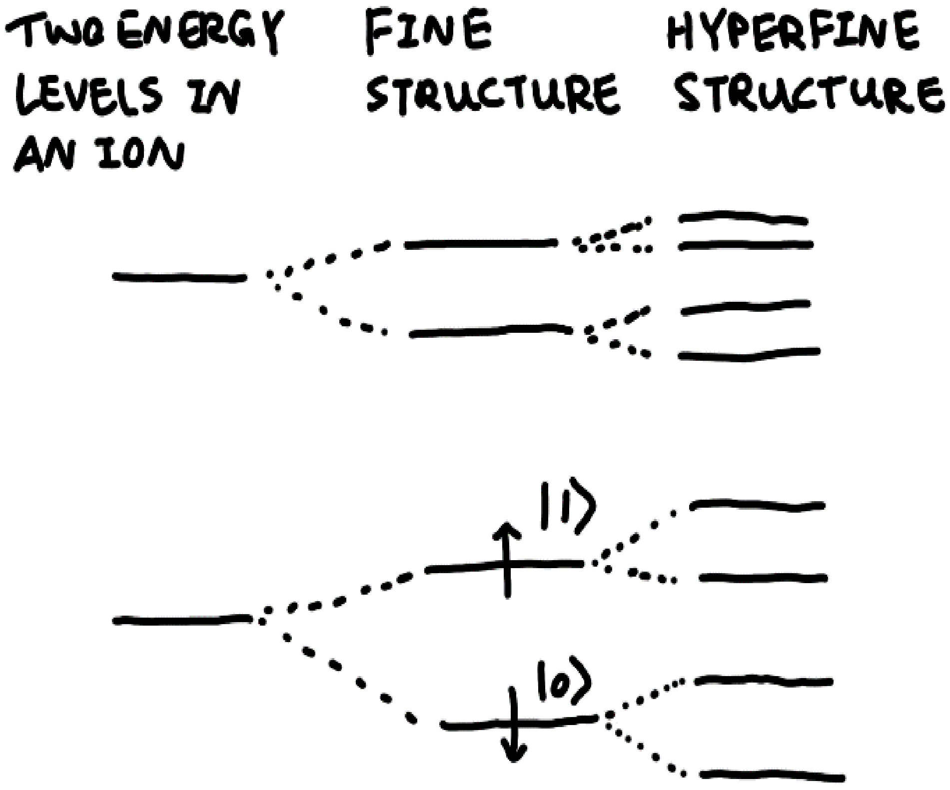
https://quantumoptics.at/en/mobile/en/news/72-scalable-multiparticle-entanglement-of-trapped-ions.html



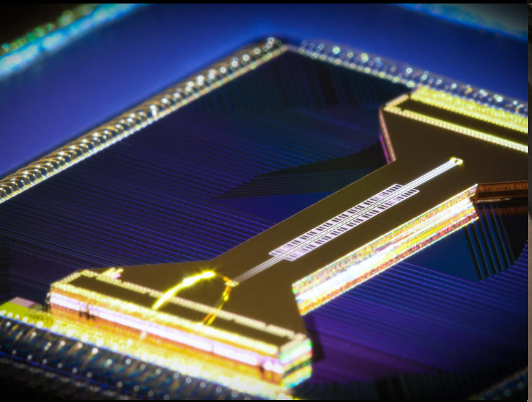
https://sciencenode.org/spotlight/nobel-prize-goes-quantum-computing-pioneers.php

Trapped Ion

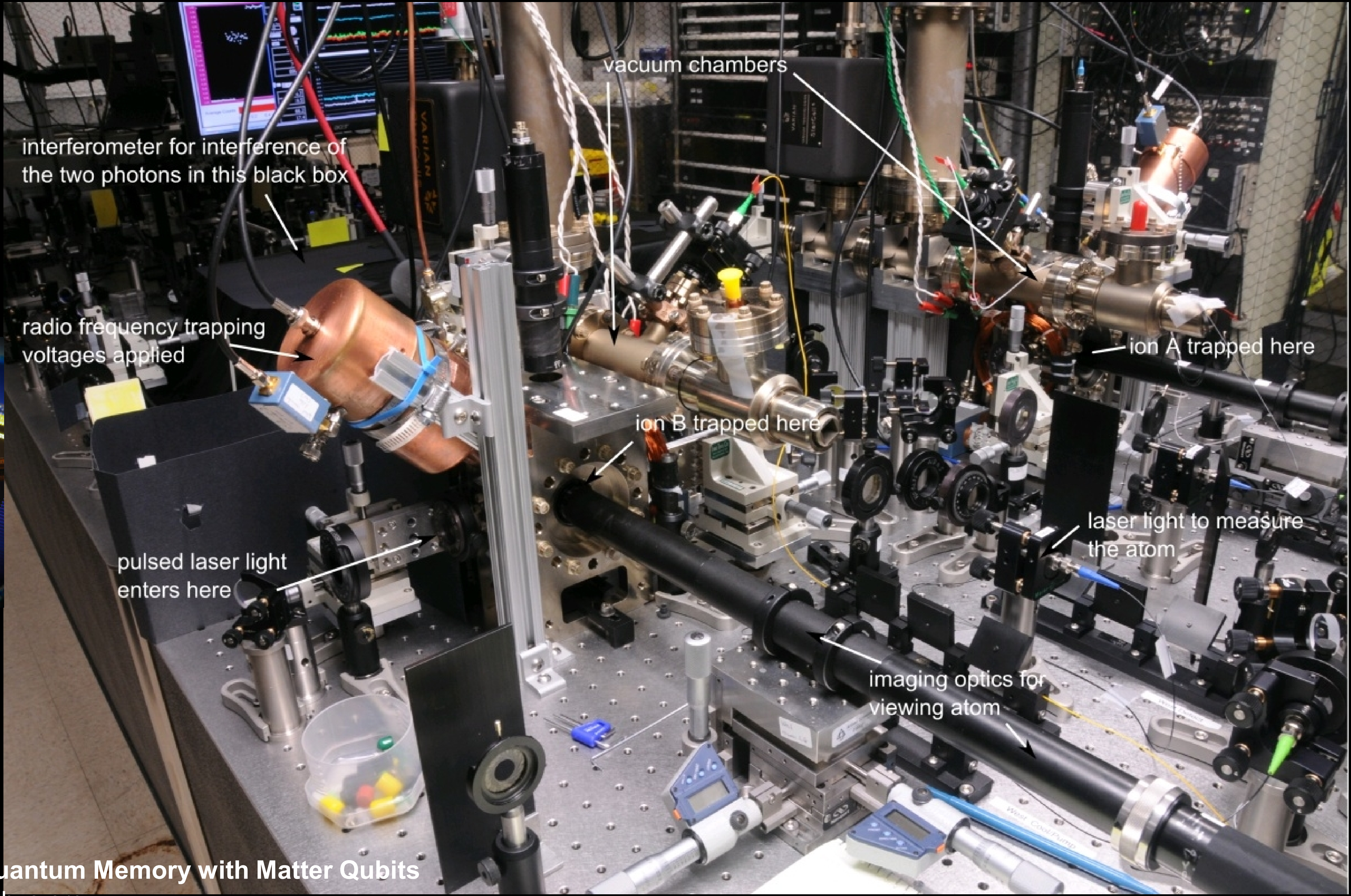

ELECTRODS CREATING E-M FIELDS,
TRAPING IONS IN A LINE
FOCUSED
LASER BEAMS


TWO ENERGY
LEVELS IN
AN ION
FINE
STRUCTURE
HYPERFINE
STRUCTURE
|1⟩
|0⟩

Trapped Ion

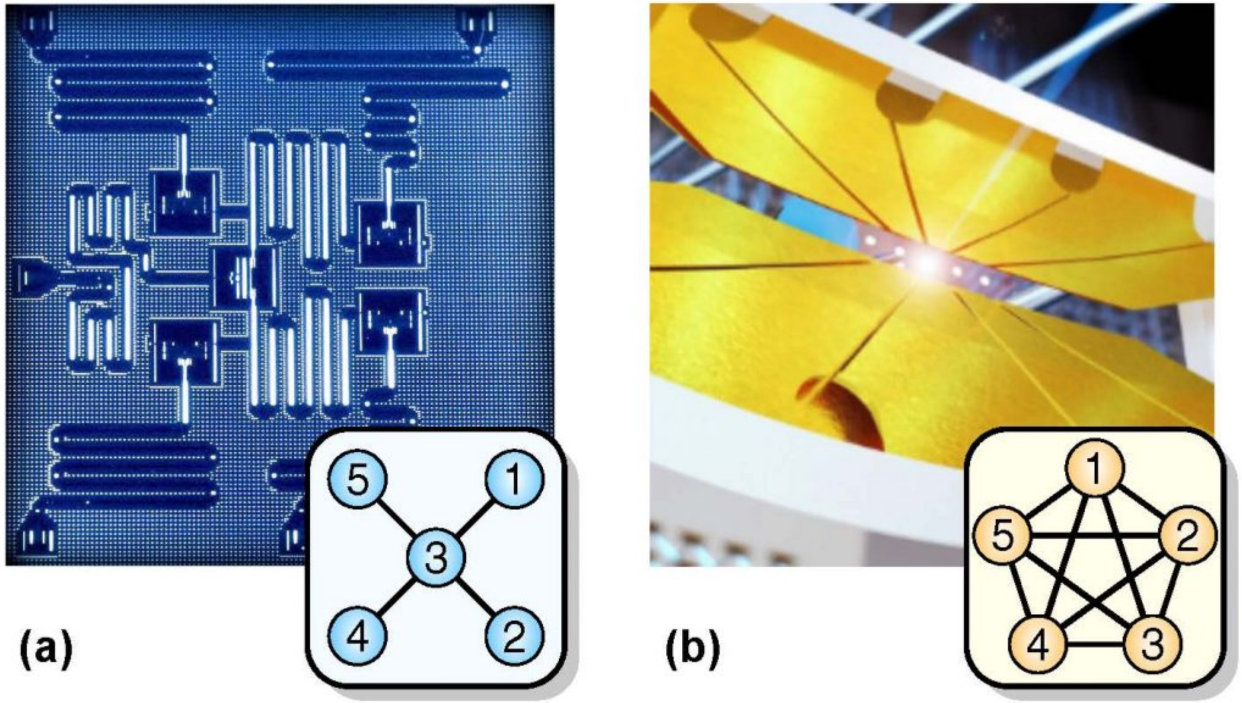


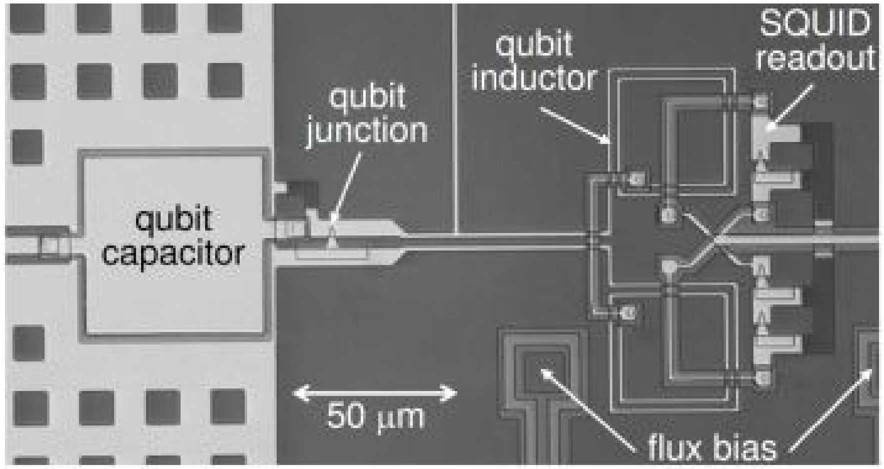
Honeywell on-chip ion trap



Physicists Demonstrate Quantum Memory with Matter Qubits
July 3, 2009 By Lisa Zyga, Phys.org

Superconducting quantum circuits

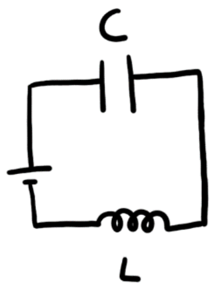


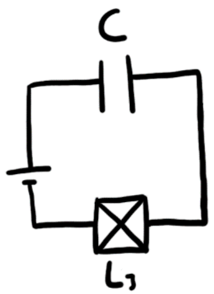


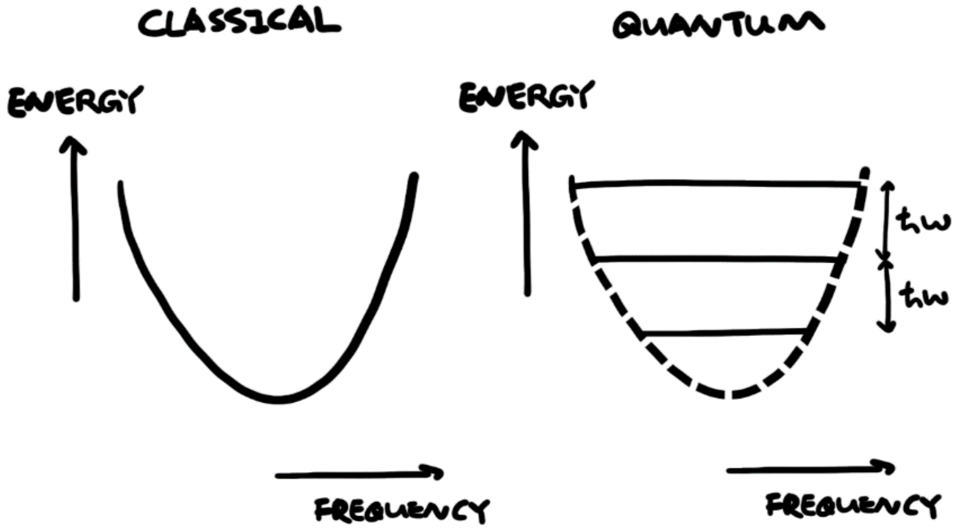
John Martinis -> Google

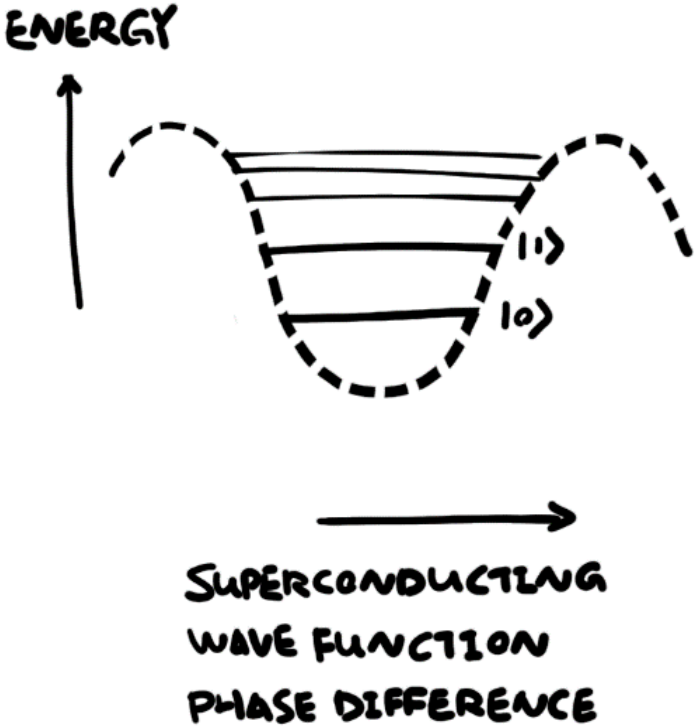
http://iontrap.umd.edu/

Superconductors vs. Trapped Ions





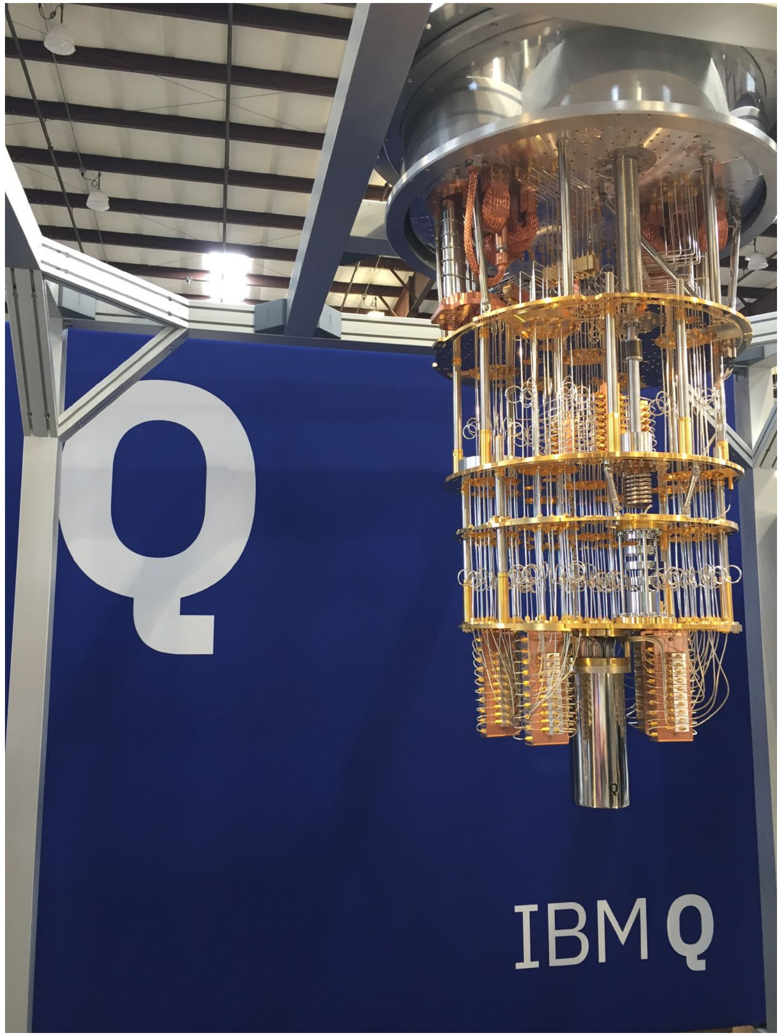


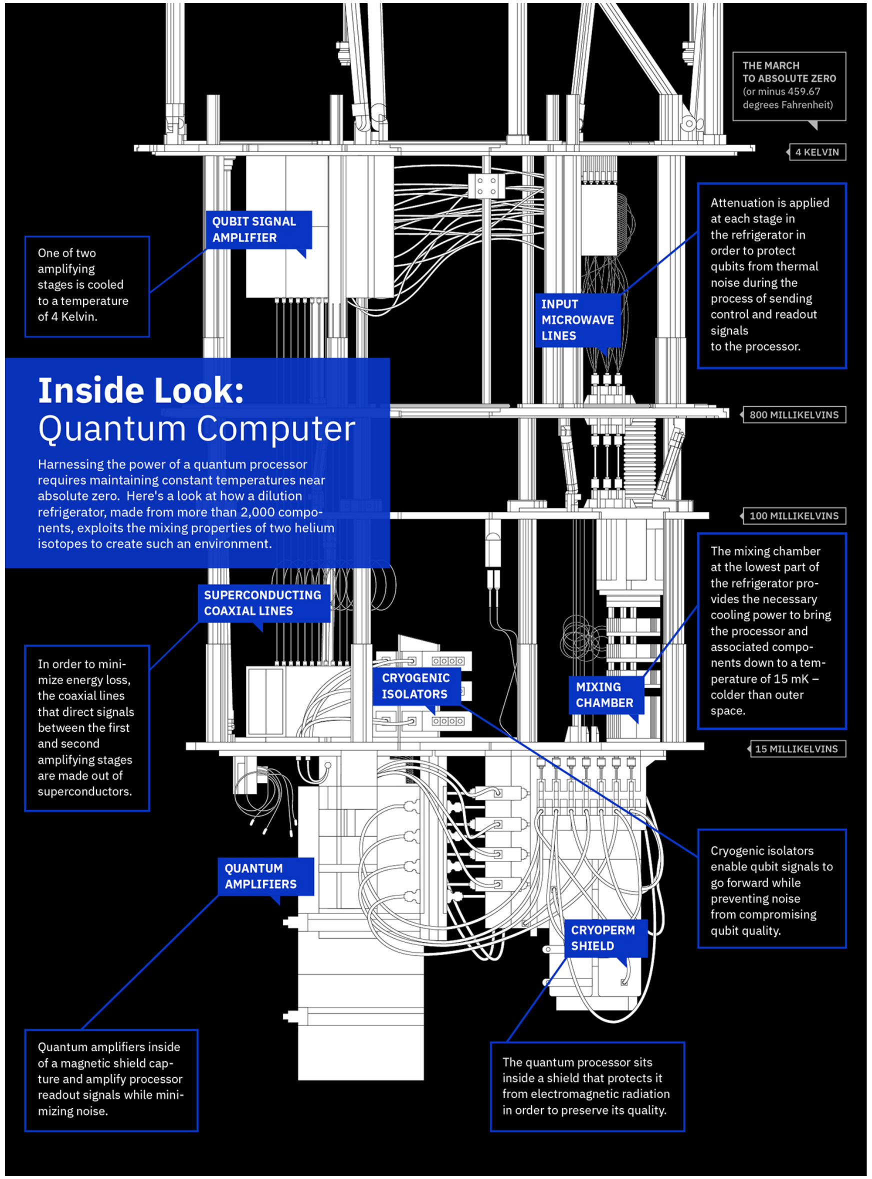


Classical to quantum mechanical:

1. effective length of the circuit is smaller than the electron scattering length in the circuit;
2. temperature is low enough: $kT < \hbar\omega$, where k is the Boltzmann constant, T is the temperature and $\omega = \sqrt{LC}$ is the natural frequency of the circuit.

Dilution refrigerators



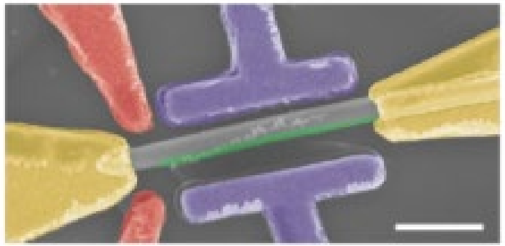


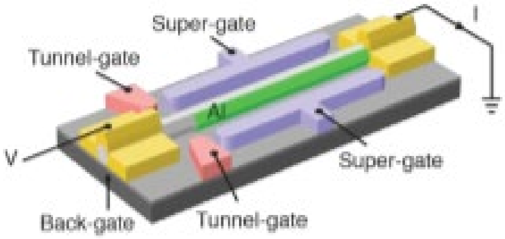
http://www.research.ibm.com/ibm-q/learn/what-is-quantum-computing/

Topological quantum computer

Majorana Fermions – particle equals anti-particle
Fractional quantum Hall conductance
Low temperature in magnetic field

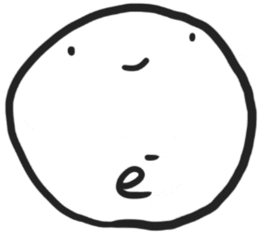
https://arxiv.org/pdf/cond-mat/0412343.pdf





Quantized Majorana Conductance

https://www.nature.com/articles/nature26142

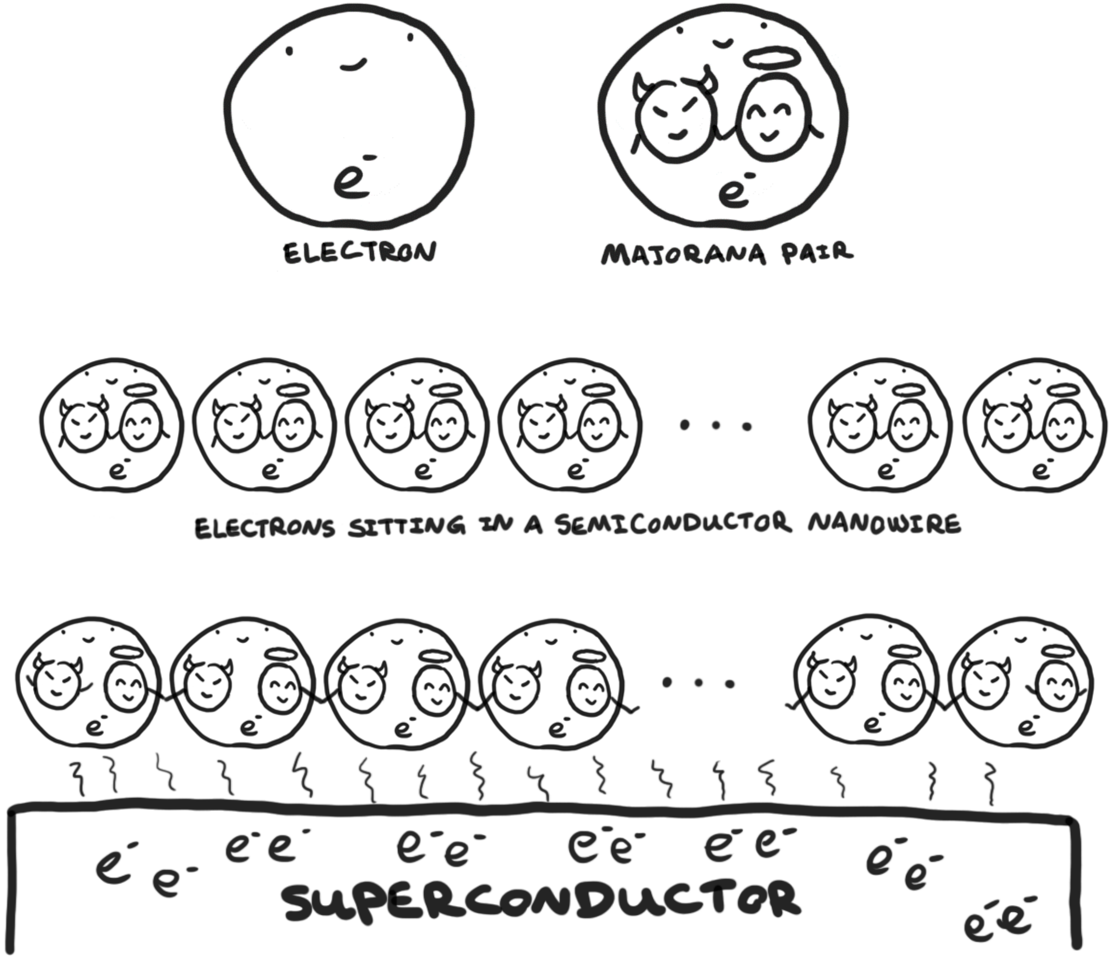


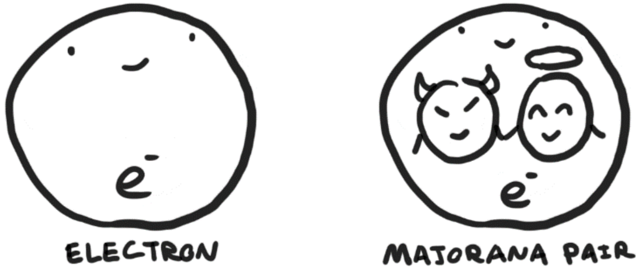
ELECTRON

MAJORANA PAIR

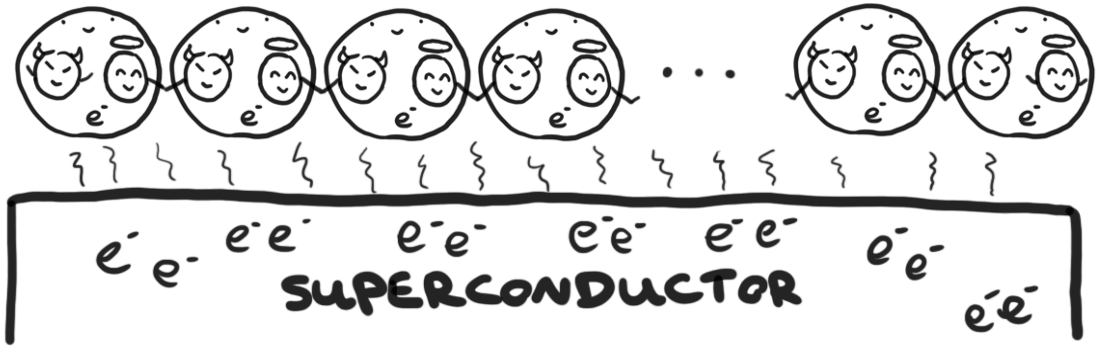
ELECTRON MAJORANA PAIR

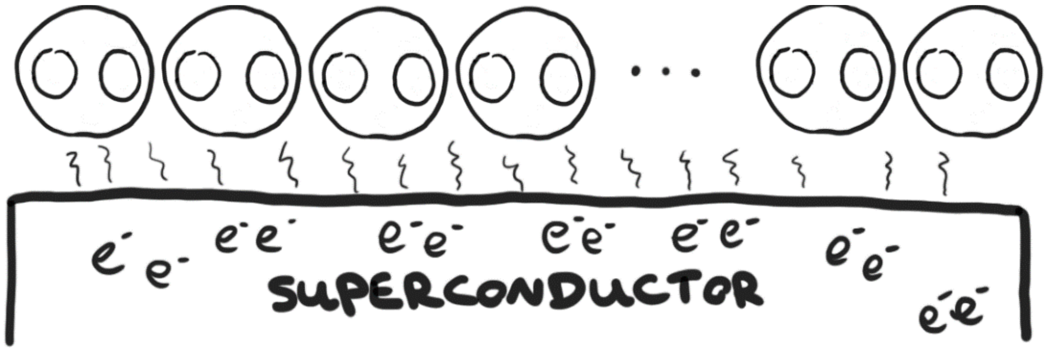
ELECTRONS SITTING IN A SEMICONDUCTOR NANOWIRE

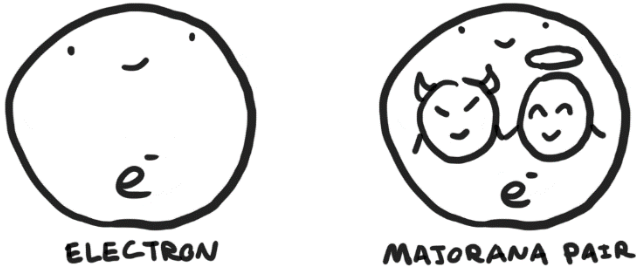

ELECTRON
MAJORANA PAIR
ELECTRONS SITTING IN A SEMICONDUCTOR NANOWIRE
SUPERCONDUCTOR

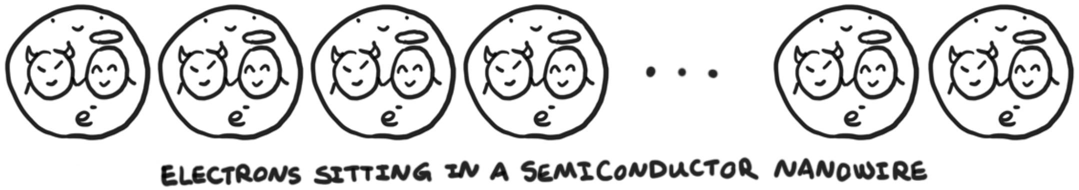


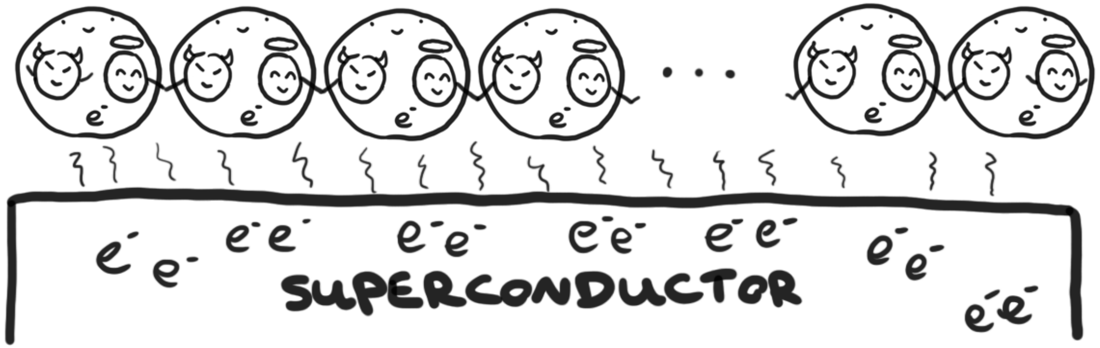
ELECTRONS SITTING IN A SEMICONDUCTOR NANOWIRE

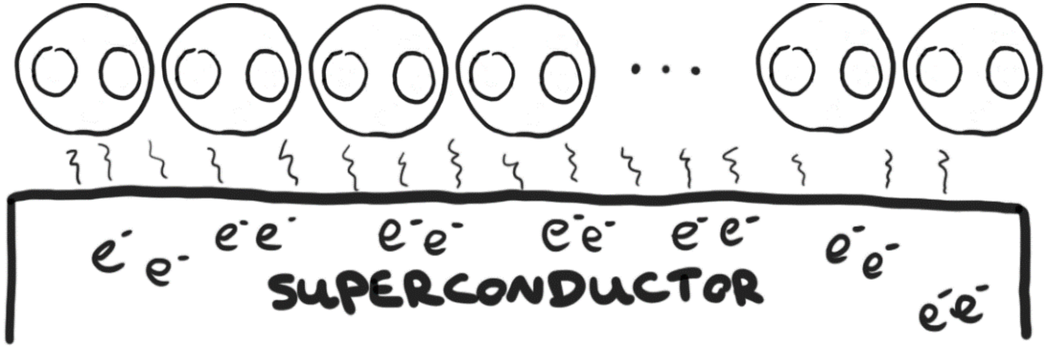


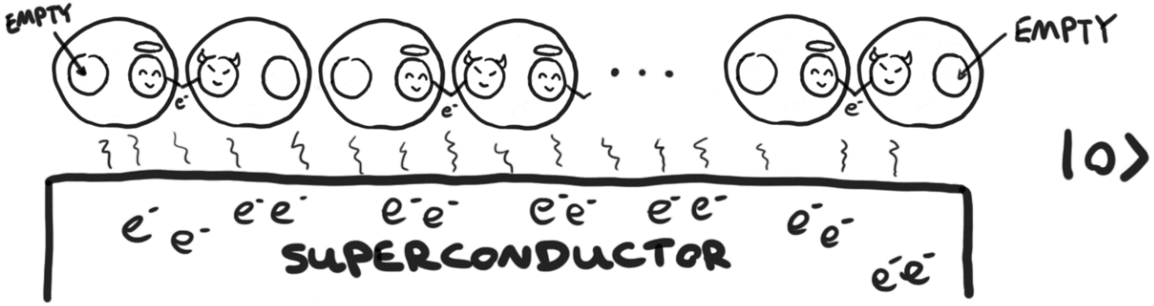


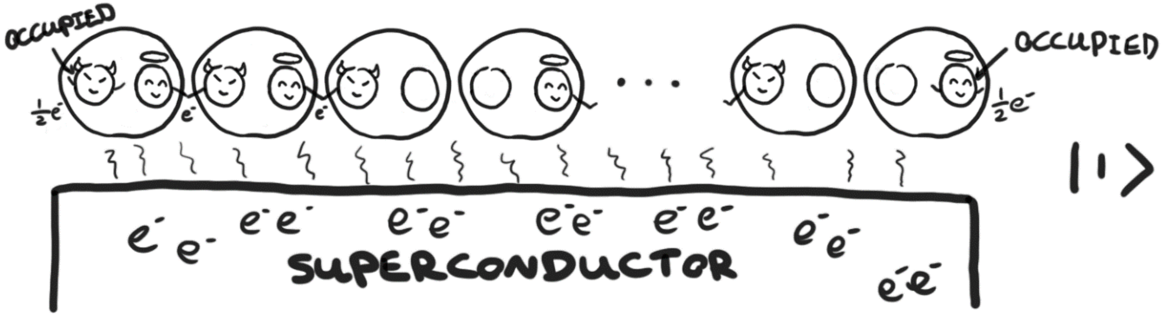

ELECTRON
MAJORANA PAIR

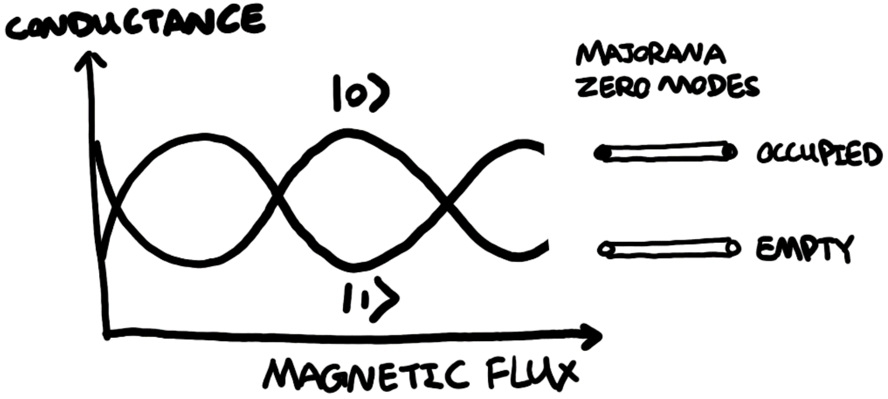

ELECTRONS SITTING IN A SEMICONDUCTOR NANOWIRE


SUPERCONDUCTOR


SUPERCONDUCTOR


EMPTY
EMPTY
SUPERCONDUCTOR
|0⟩


OCCUPIED
OCCUPIED
SUPERCONDUCTOR
|1⟩

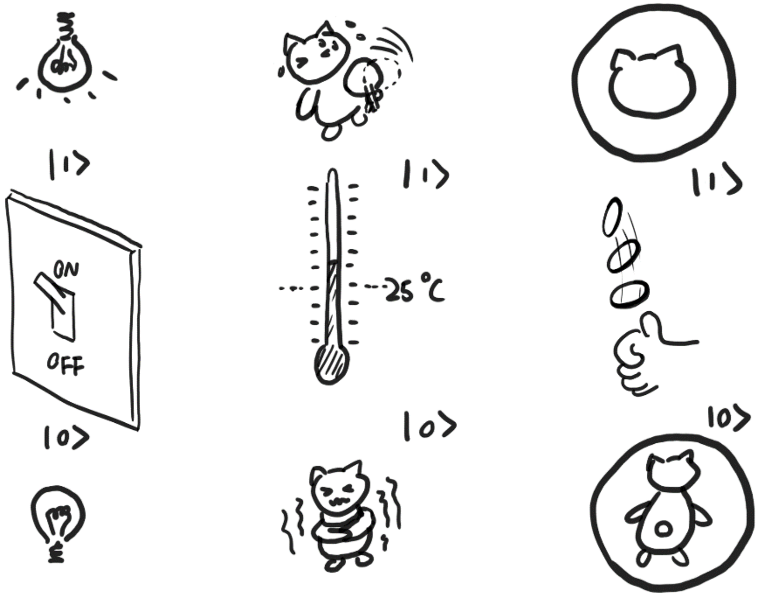

CONDUCTANCE
|0⟩
|1⟩
MAGNETIC FLUX
MAJORANA ZERO MODES
OCCUPIED
EMPTY

Q#

- Installing QDK
- https://marketplace.visualstudio.com/items?itemName=quantum.DevKit
- Visual Studio or Visual Studio Code
- Jupyter Notebook katas

States – classical bits

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



MULTIPLE CLASSICAL BITS OF "0"s & "1"s.

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$|00\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}.$$

$$|01\rangle = \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix},$$

$$|10\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix},$$

$$|11\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}.$$

Math insert - Tensor product---

How does tensor product $\otimes$ work?

$$\begin{pmatrix}x_0\\x_1\end{pmatrix} \otimes \begin{pmatrix}y_0\\y_1\end{pmatrix} = \begin{pmatrix}x_0\begin{pmatrix}y_0\\y_1\end{pmatrix}\\x_1\begin{pmatrix}y_0\\y_1\end{pmatrix}\end{pmatrix} = \begin{pmatrix}x_0y_0\\x_0y_1\\x_1y_0\\x_1y_1\end{pmatrix}$$

and

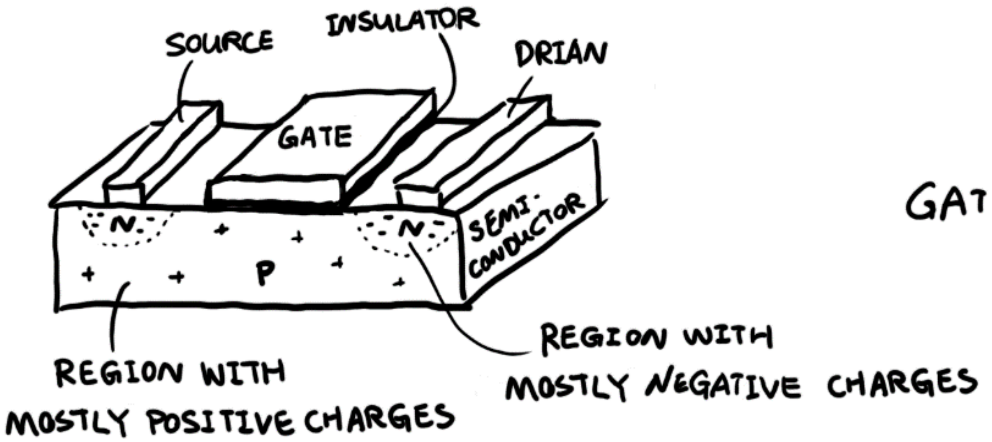
$$\begin{pmatrix}x_0\\x_1\end{pmatrix} \otimes \begin{pmatrix}y_0\\y_1\end{pmatrix} \otimes \begin{pmatrix}z_0\\z_1\end{pmatrix} = \begin{pmatrix}x_0y_0z_0\\x_0y_0z_1\\x_0y_1z_0\\x_0y_1z_1\\x_1y_0z_0\\x_1y_0z_1\\x_1y_1z_0\\x_1y_1z_1\end{pmatrix}$$

and so on.

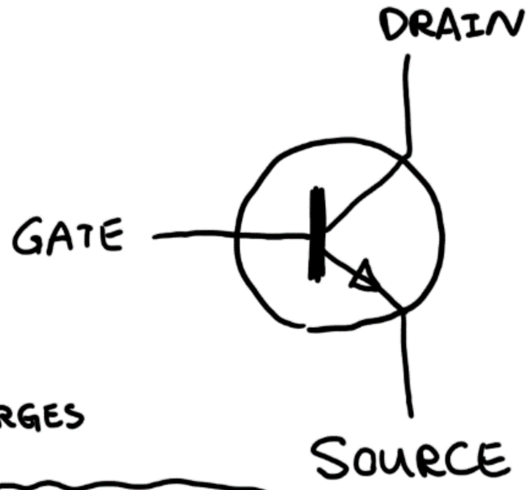
For example, the number 4 can be represented with a three-bit string 100. We can write

$$|4\rangle = |100\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\1\\0\\0\\0\end{pmatrix}.$$

MATERIAL CROSS-SECTION



CIRCUIT SYMBOL



IN CLASSICAL COMPUTERS, "0"s & "1"s ARE ACHIEVED USING $\boxed{\text{TRANSISTORS.}}$ THEY ARE MADE OF LAYERS OF CONDUCTORS, SEMI-CONDUCTORS AND INSULATORS. DEPENDING ON THE VOLTAGE APPLIED TO THE GATE, ELECTRIC CURRENT CAN FLOW FROM SOURCE TO DRAIN, OR NOT, THUS, ACTING LIKE A SWITCH FLIPPING BETWEEN "ON" OR "OFF".

$|1\rangle$ $|0\rangle$

THE BUILDING BLOCKS OF QUANTUM COMPUTERS ARE VERY DIFFERENT.

Quantum bits – qubits

A SPINNING COIN IS LIKE A QUBIT. EITHER LANDING ON "HEADS" OR "TAILS" IS POSSIBLE — "HEADS" AND "TAILS" ARE IN SUPERPOSITION.

$$|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix} = a|0\rangle + b|1\rangle$$

$$|a|^2 + |b|^2 = 1$$

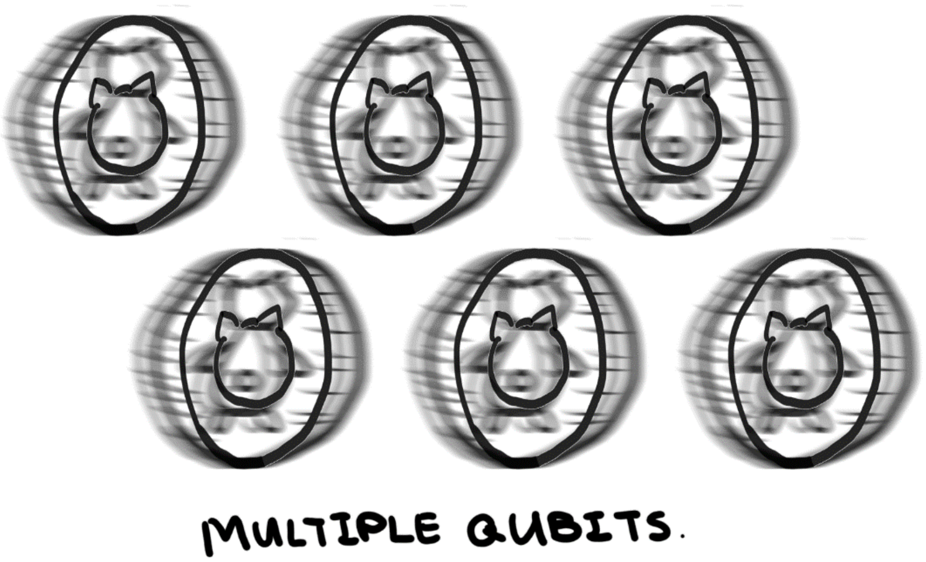
$$|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix}$$

$$= \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$$

$$= ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle$$

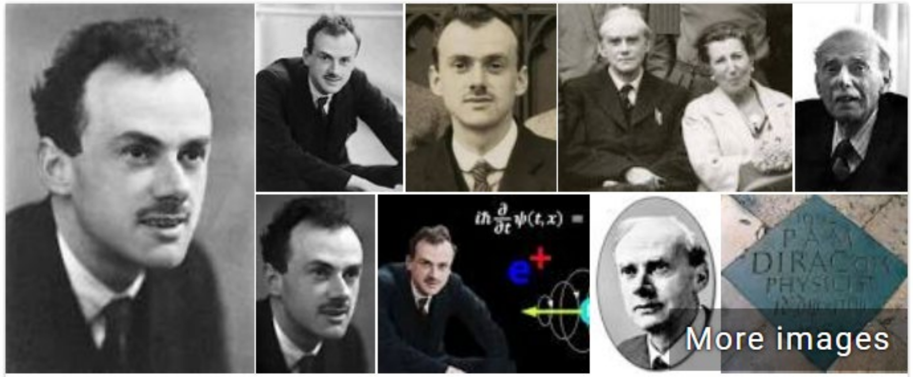
$$|ac|^2 + |ad|^2 + |bc|^2 + |bd|^2 = 1$$

Superposition



Superposition of states is the fundamental factor that's making quantum computing powerful. Because while a classical bit can only be in either $|0\rangle$ or $|1\rangle$, a qubit can be in a state where $|0\rangle$ and $|1\rangle$ coexist - a complex linear combination between $|0\rangle$ and $|1\rangle$. Thus, if we make a computing system out of this quantum phenomenon, we can have a single qubit that contains information that two classical bits would be needed. With N qubits, the system can compute 2^N classical bits of information.

Dirac notation and wavefunction



Paul Dirac

Physicist

Paul Adrien Maurice Dirac OM FRS was an English theoretical physicist who is regarded as one of the most significant physicists of the 20th century. Dirac made fundamental contributions to the early development of both quantum mechanics and quantum electrodynamics. Wikipedia

Born: August 8, 1902, Bristol, United Kingdom

Died: October 20, 1984, Tallahassee, FL

Field: Theoretical physics

Spouse: Margit Wigner (m. 1937–1984)

Schrödinger equation has the form of a wave equation

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\Psi(\boldsymbol{r},t) + V(\boldsymbol{r},t)\Psi(\boldsymbol{r},t) = i\hbar\frac{\partial\Psi(\boldsymbol{r},t)}{\partial t}}$$

Therefore the solution is a linear combination Of all the possible wavefunctions

$$\psi(x) = \sum_i c_i\phi_i(x)$$

$$\int_{-\infty}^{+\infty} \phi_j{}^*(x)\ \psi(x)dx = \sum_i c_i \int_{-\infty}^{+\infty} \phi_j(x)^*\ \phi_i(x)dx = c_j\ .$$

In Dirac notation, $|\psi\rangle = \sum_i c_i|\phi_i\rangle$, where $c_j = \langle\phi_j|\psi\rangle$.

$|\Psi\rangle$ denotes "the state with wavefunction" $\Psi(\boldsymbol{r},t)$

$$\Psi^*(\boldsymbol{r},t) = \langle\Psi|$$

$$\int_{-\infty}^{+\infty} \phi^*(x)\,\psi(x)\,dx \equiv \langle\phi|\psi\rangle$$

A qubit only has two "wavefunctions"

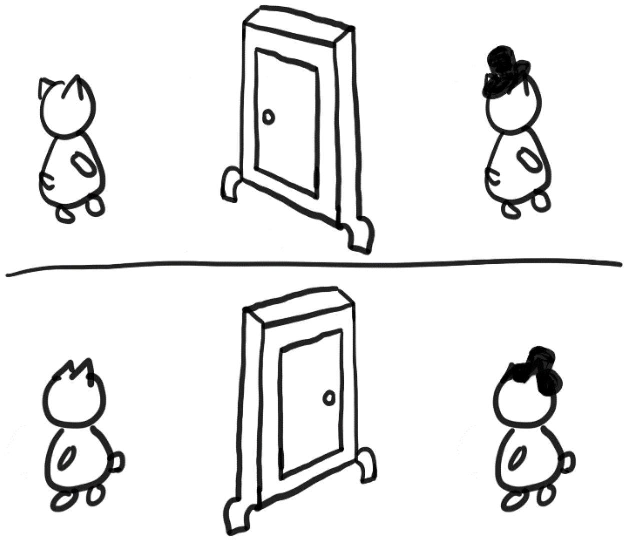
$$\psi(x) = \sum_i c_i \phi_i(x)$$

Nature

$$|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix} = a|0\rangle + b|1\rangle$$

Computing

Gates



manipulate qubit states (vectors)
through matrix multiplications

unitarity $U^{\dagger}U = I$

So that it is reversible and probabilities add up to 1

Math insert – unitary, adjoint or Hermitian conjugate ---

In math, unitarity means $U^{\dagger}U = I$, where I is the identity matrix and the "†" symbol (reads "dagger") means adjoint or Hermitian conjugate of matrix U. It can be further written as $U^{\dagger} = (U^{*})^{T} = (U^{T})^{*}$, where "*T*" denotes transpose and "*" complex conjugate:

$$\begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_N \end{pmatrix}^T = \begin{pmatrix} U_1 & U_2 & \dots & U_N \end{pmatrix}$$

and if $a = a_0 + ia_1$, then $a^{*} = a_0 - ia_1$ by definition. Therefore,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{\dagger} = \begin{pmatrix} a^{*} & c^{*} \\ b^{*} & d^{*} \end{pmatrix}.$$

CNOT

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Math insert - Matrix multiplication --

Gates are N by N matrices that multiply to state with 2^N vector elements. They follow the rules such that

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix},$$

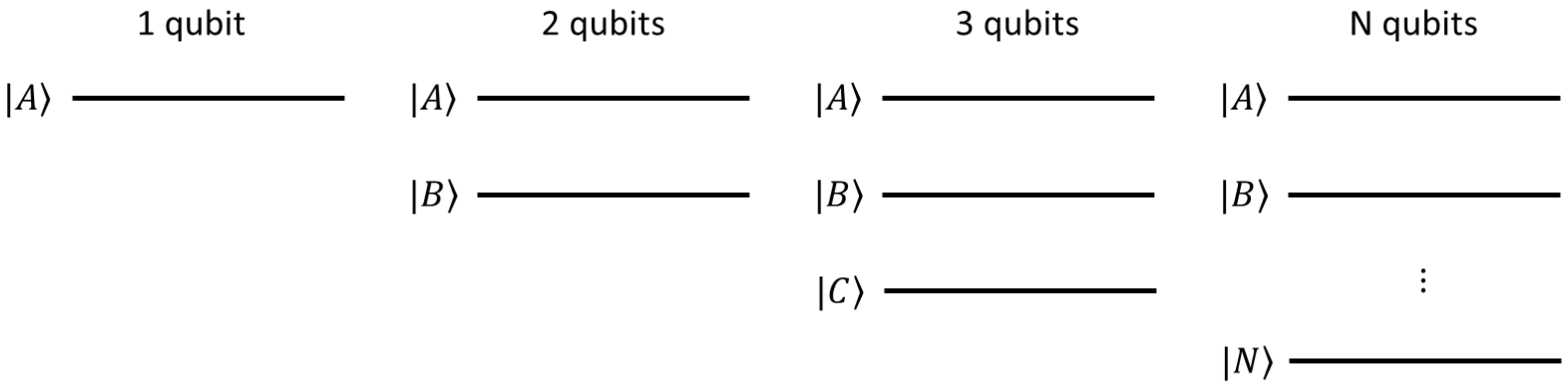
$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{pmatrix},$$

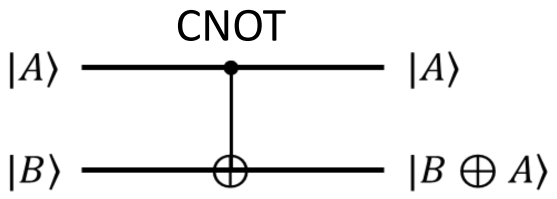
and so on.

$$CNOT|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = |11\rangle\,.$$

Similarly, $C|00\rangle = |00\rangle, C|01\rangle = |01\rangle$ and $C|11\rangle = |10\rangle$.

Circuit representation





Target B controlled by A

Hadamard H

$$H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

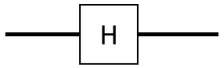
Hadamard H

$$H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$H|0\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

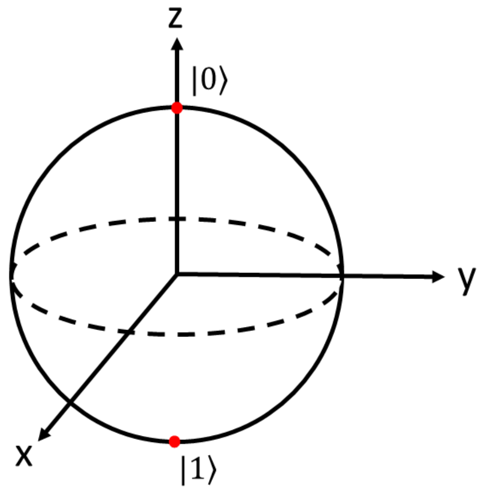
$$= \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \equiv |+\rangle$$

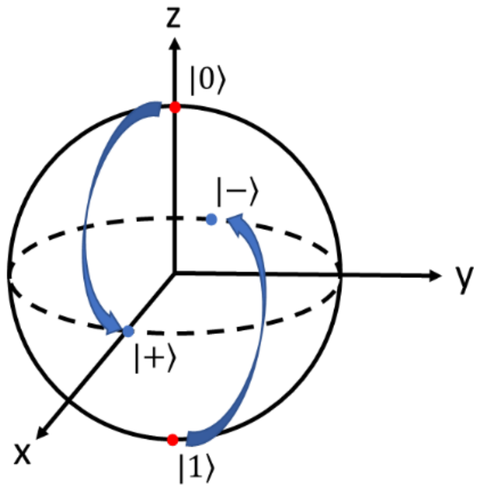


$$H|1\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

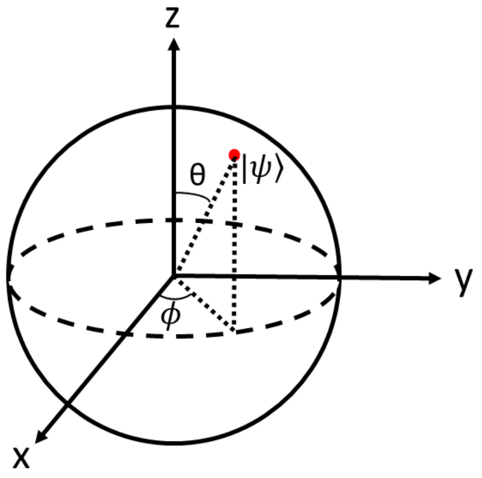
$$= \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \equiv |-\rangle \,.$$

Bloch sphere





H gate

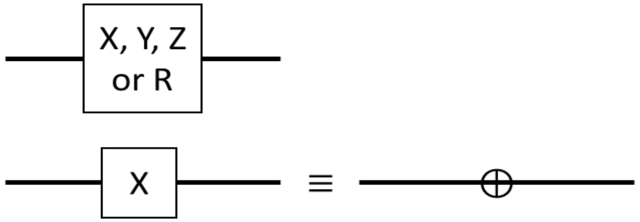


Arbitrary state

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{-i\phi}\sin\frac{\theta}{2}|1\rangle$$

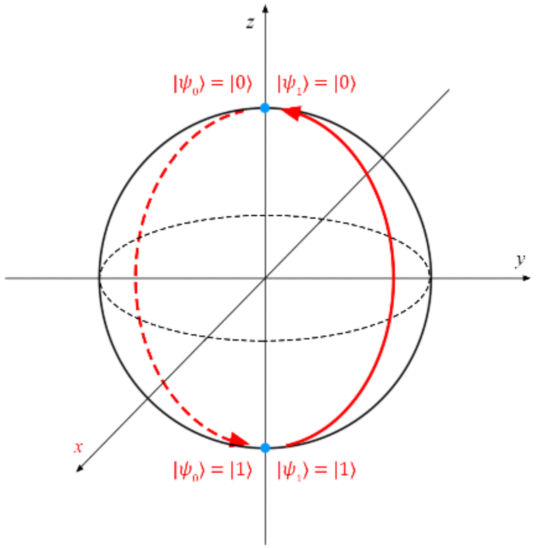
the states $|0\rangle$ and $|1\rangle$ are just two special cases with θ = 0° and 180°, respectively.

Pauli gates



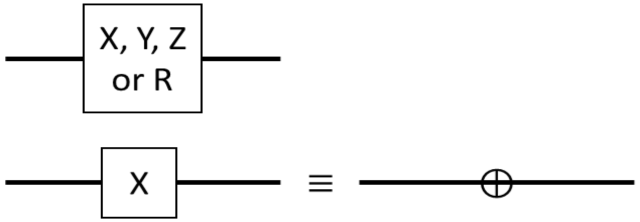
$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$



Pauli X

Pauli gates

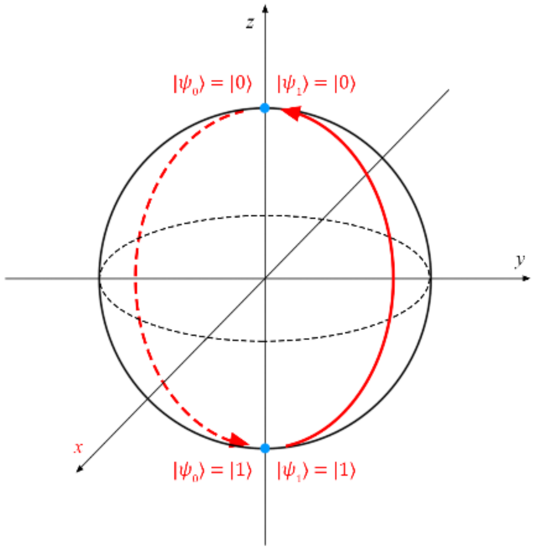


$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

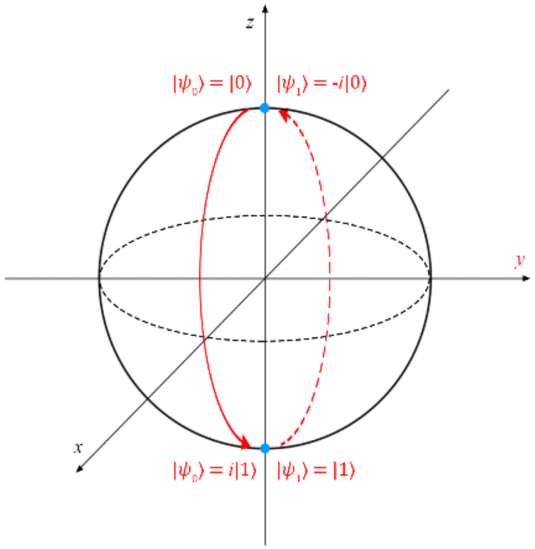
$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$X\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$

$$Y\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = i\begin{pmatrix} -\beta \\ \alpha \end{pmatrix}$$

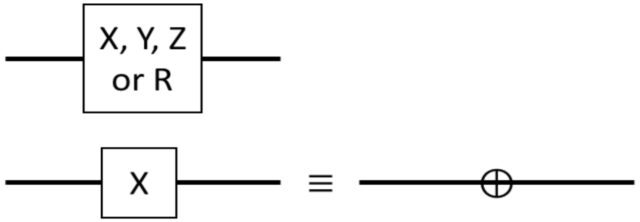


Pauli *X*



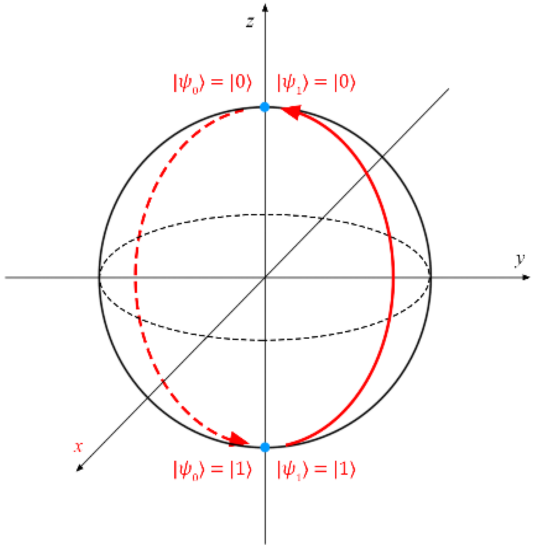
Pauli *Y*

Pauli gates

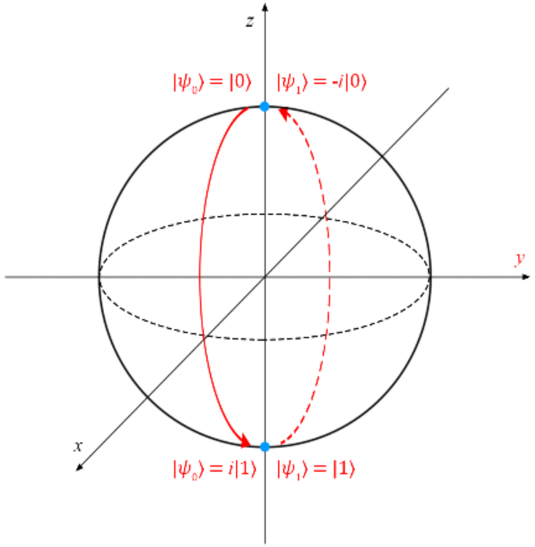


$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

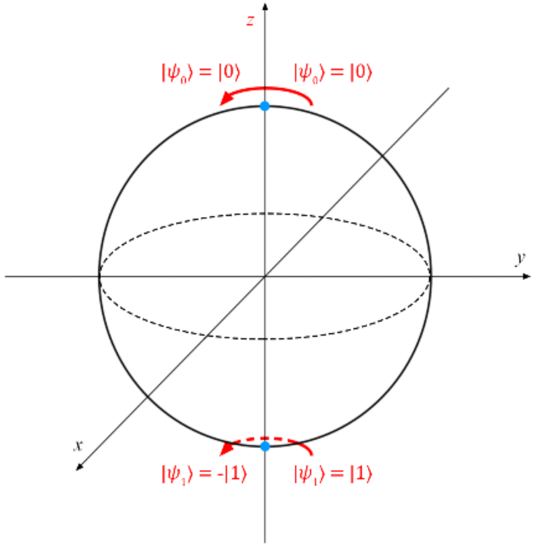
$$X \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix} \qquad Y \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = i \begin{pmatrix} -\beta \\ \alpha \end{pmatrix} \qquad Z \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ -\beta \end{pmatrix}$$



Pauli X



Pauli Y



Pauli Z

General rotation

In general, rotation gates, R, about an axis can be described by the angles ϕ and θ:

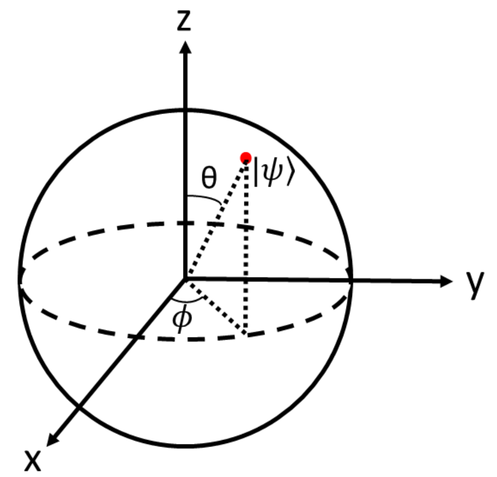
$$R_z(\phi) = \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix},$$

$$R_y(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix},$$

and

$$R_x(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

$$= R_z\left(\frac{\pi}{2}\right) R_y(\theta) R_z\left(-\frac{\pi}{2}\right).$$



General rotation

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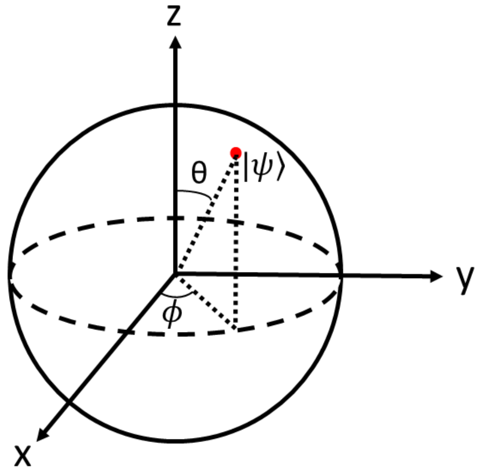
and

$$R_x(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

$$= R_z\left(\frac{\pi}{2}\right) R_y(\theta) R_z\left(-\frac{\pi}{2}\right).$$

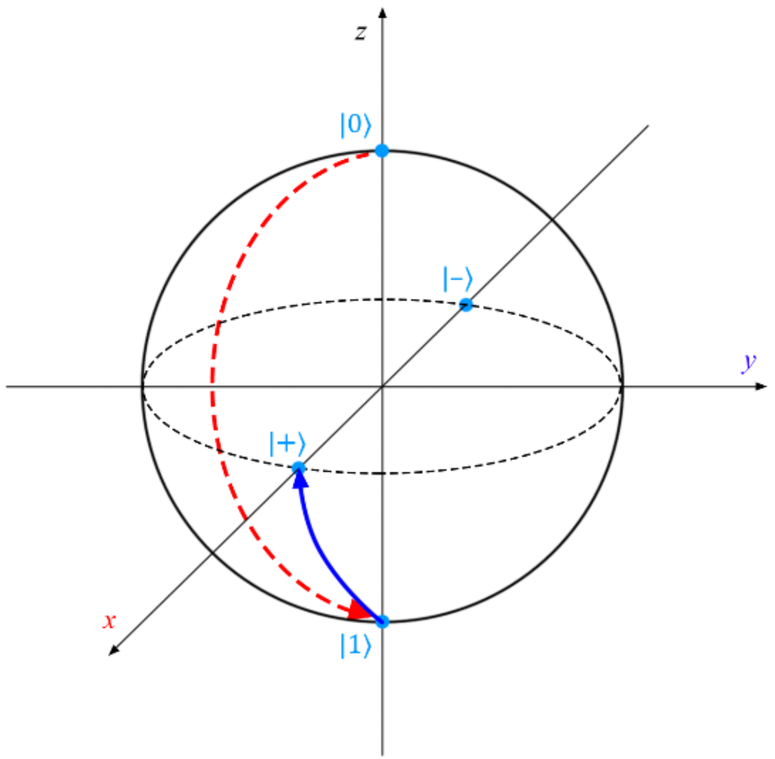


In fact, any arbitrary single quantum logic gate can be decomposed into a series of rotation matrices:

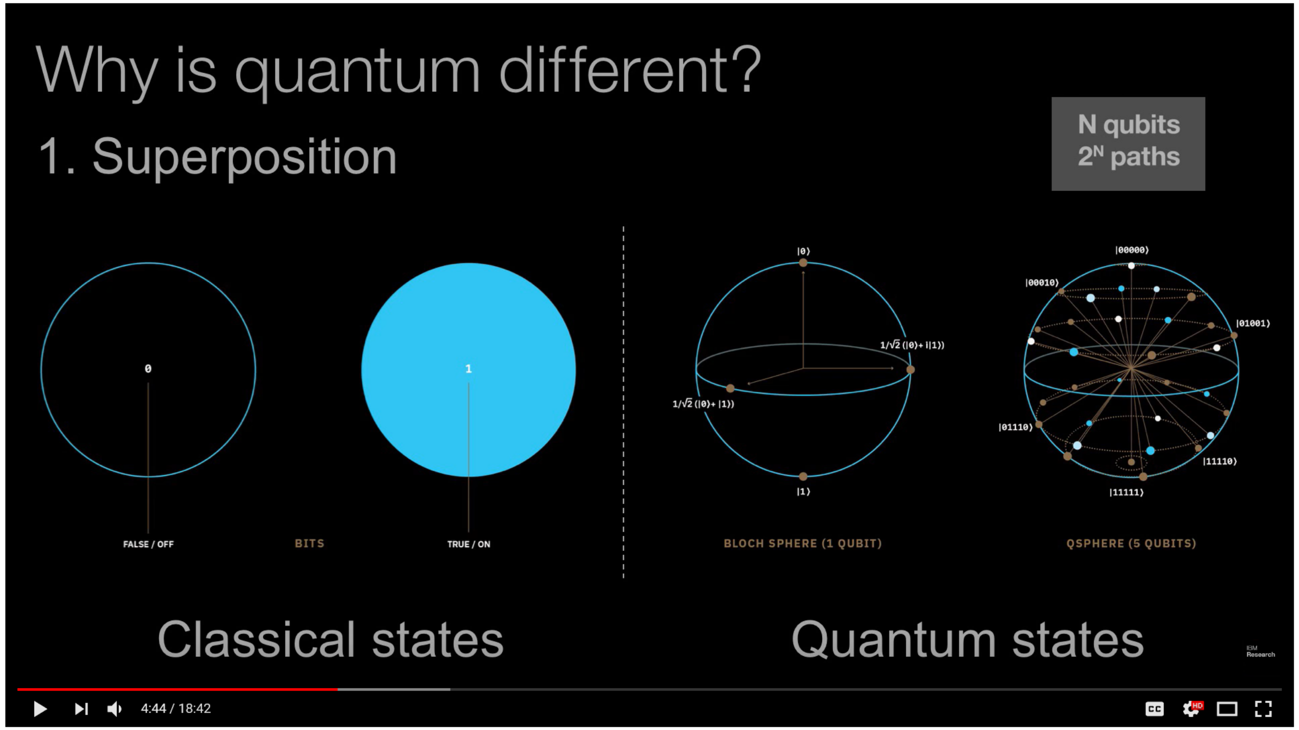
$$U = e^{i\gamma} \begin{bmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{bmatrix} \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

with the only constraint on the gate being unitary. Here, $e^{i\gamma}$ is a global phase shift that can be added without affecting the behavior.

Hadamard revisit



$H|0\rangle = |+\rangle$



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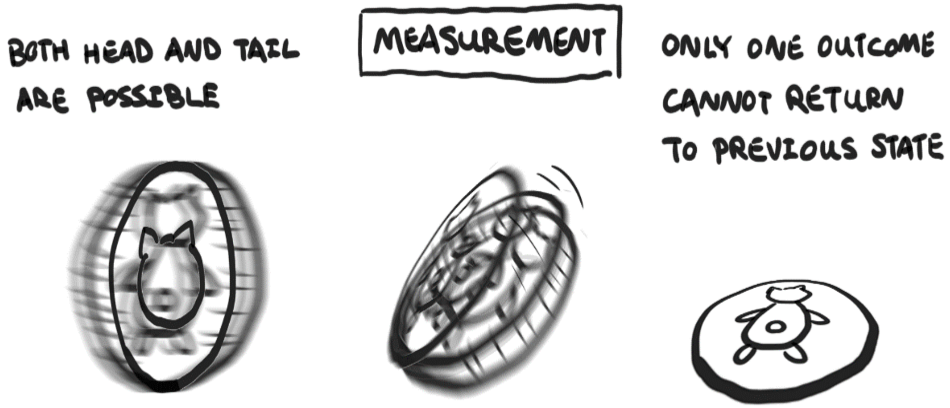
Measurement – not a gate



Not reversible

$$|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

$$P = |c_{00}|^2 + |c_{01}|^2$$

If first qubit is 0

$$|\psi'\rangle = \frac{c_{00}|00\rangle + c_{01}|01\rangle}{\sqrt{P}}$$

After measurement

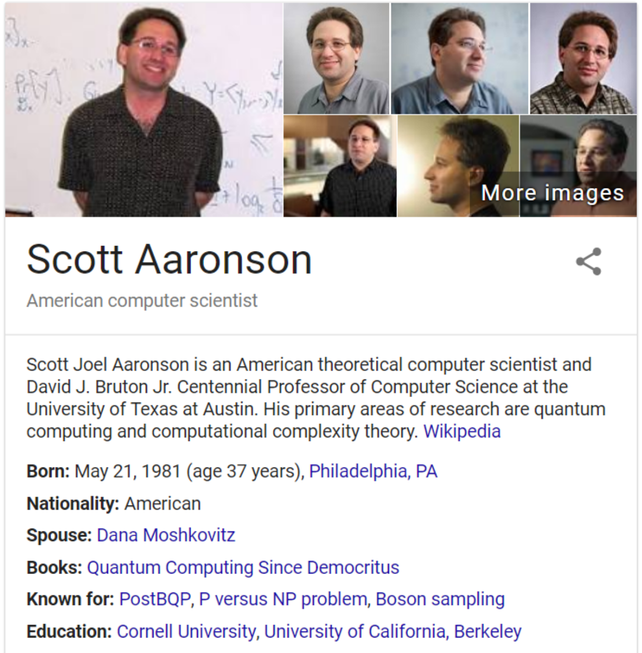
Measurement

If we use the wavefunction approach, we can derive the value we'd expect to measure for a large number of measurements of a given observable, *M*. The expectation value can be obtained as

$$\langle M \rangle = \langle \psi | M | \psi \rangle = \sum_j m_j \, |c_j|^2 \, ,$$

where m_j is each measurement result of M, and $|c_j|^2 = P(m_j)$ is the probability of getting result m_j. Obtaining m_j leaves the system in the state $|\psi_j\rangle$. This unavoidable disturbance of the system caused by the measurement process is often described as a "collapse," a "projection" or a "reduction" of the wavefunction.

Generalized probability theory -> forget about wavefuntions, just look at probability



$$\sum_i p_i = 1$$

1-norm
Classical

$$\sum_i |a_i|^2 = 1$$

2-norm
Quantum mechanical

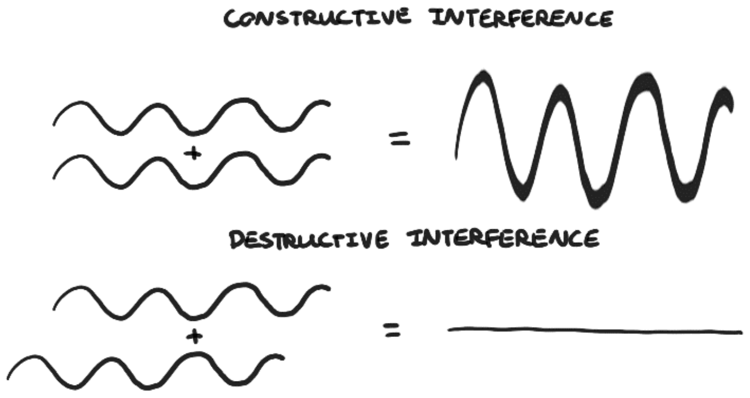
2-norm Vs 1-norm
https://www.scottaaronson.com/democritus/lec9.html

Amplitude can be positive, negative or complex

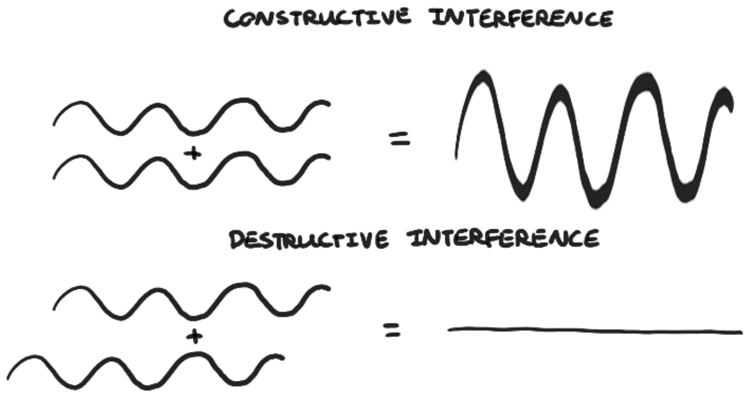
To read more rigorous mathematical derivations of the axioms in modern quantum theory:

- https://arxiv.org/abs/quant-ph/0101012
- https://arxiv.org/abs/1011.6451
- https://arxiv.org/abs/quant-ph/0104088

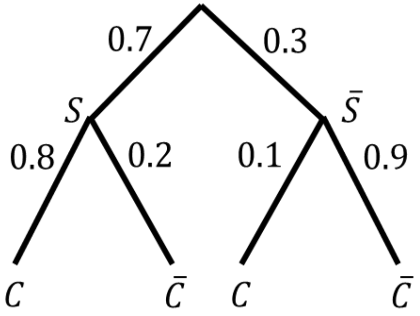
Interference

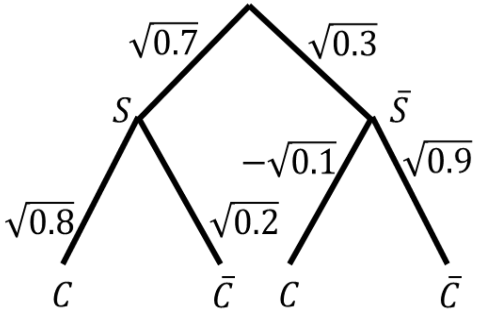


Interference



Classical





Quantum mechanical

$P(C) = 0.7 \times 0.8 + 0.3 \times 0.1 = 0.59$, or 59%.

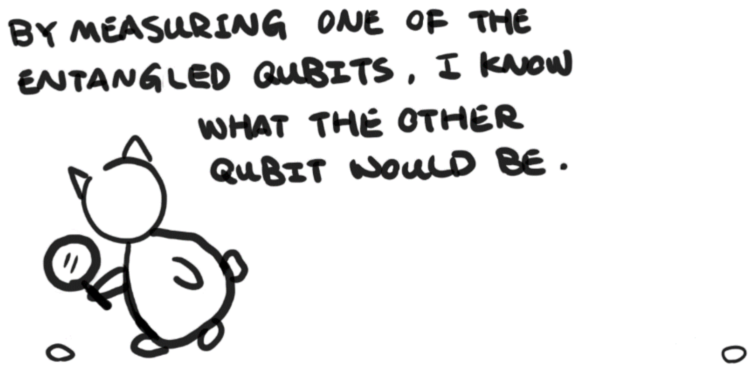
$a_c = \sqrt{0.7} \times \sqrt{0.8} - \sqrt{0.3} \times \sqrt{0.1}$

$P(C) = |a_c|^2 \approx 0.548$, or 54.8%.

Entanglement

Bell states

$|\varphi^{\pm}\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$ and $|\phi^{\pm}\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$



Take $|\phi^{+}\rangle$ as an example, upon measuring the first qubit, one obtains two possible results:

1. First qubit is 0, get a state $|\phi'\rangle = |00\rangle$ with probability ½.
2. First qubit is 1, get a state $|\phi''\rangle = |11\rangle$ with probability ½.

If the second qubit is measured, the result is the same as the above. This means that measuring one qubit tells us what the other qubit is.

Entanglement

Math insert – entangled states cannot be factored back to individual qubits--------------

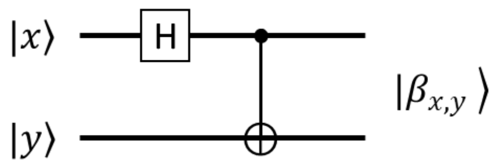
Remember in section 1.1, a two-qubit state can be obtained by doing a tensor product of two individual one-qubit states. However, a Bell state cannot be factored back into two individual qubits. For example,

$$|\phi^{\pm}\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}.$$

If we want to factor it back to two separate qubits as in $\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix}$, then this set of equations need to be simultaneously satisfied

$ac = \frac{1}{\sqrt{2}}$, $ad = 0, bc = 0$ and $bd = \frac{1}{\sqrt{2}}$. Unfortunately, it is impossible. This set of equations has no solution. It can only be 50% chance of getting $|00\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or $|11\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Creating Bell states



In	Out
$\|00\rangle$	$(\|00\rangle + \|11\rangle)/\sqrt{2} \equiv \|\beta_{00}\rangle$
$\|01\rangle$	$(\|01\rangle + \|10\rangle)/\sqrt{2} \equiv \|\beta_{01}\rangle$
$\|10\rangle$	$(\|00\rangle - \|11\rangle)/\sqrt{2} \equiv \|\beta_{10}\rangle$
$\|11\rangle$	$(\|01\rangle - \|10\rangle)/\sqrt{2} \equiv \|\beta_{11}\rangle$

Try proving this table

Greenberger – Horne – Zeilinger (GHZ) states

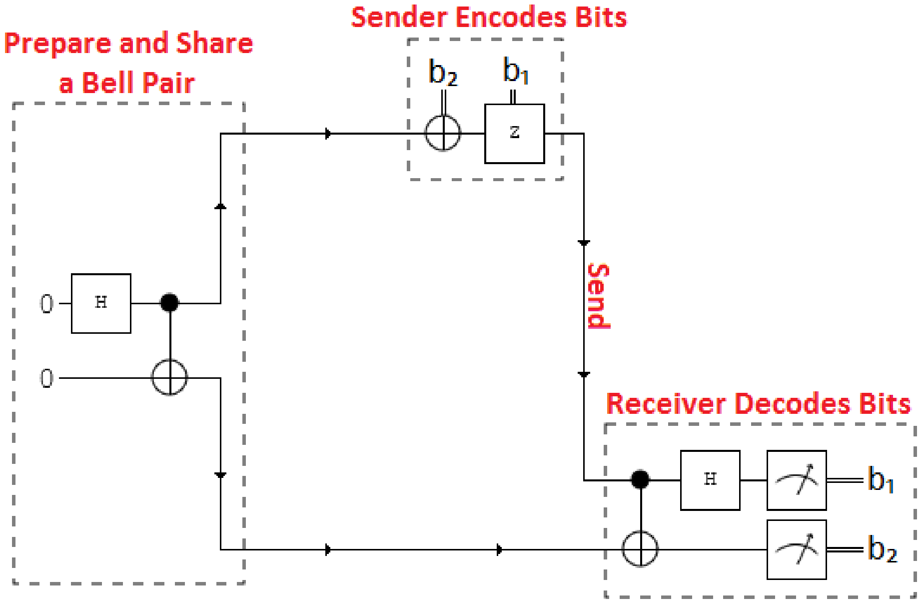
$$|GHZ\rangle_{simplest} = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

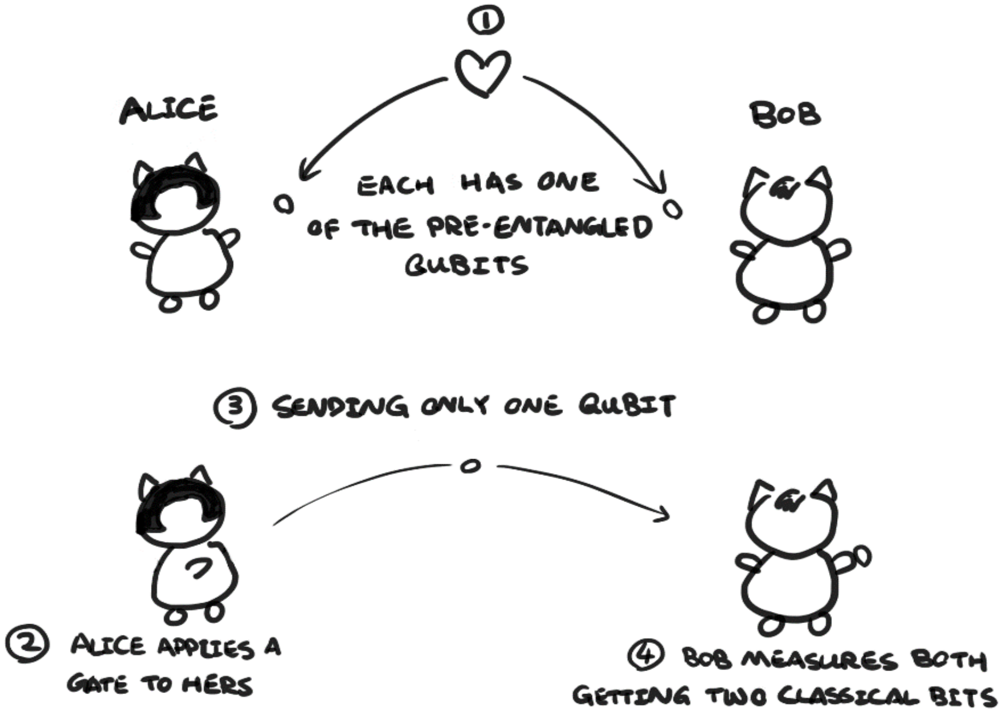
$$|GHZ\rangle_{general} = \frac{|0\rangle^{\otimes \mathrm{N}} + |1\rangle^{\otimes \mathrm{N}}}{\sqrt{2}}$$

Imagine there are N entangled qubits. Because they are correlated, by measuring one qubit, we know the result of another qubit. If N = 500, there are 2^{500} possible states in the system - more than the number of atoms in the Universe. Yet if they are all entangled, the Universe stores and calculates that amount of data simultaneously. This is the power of Nature that quantum computing utilizes.

$$|\phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Superdense coding



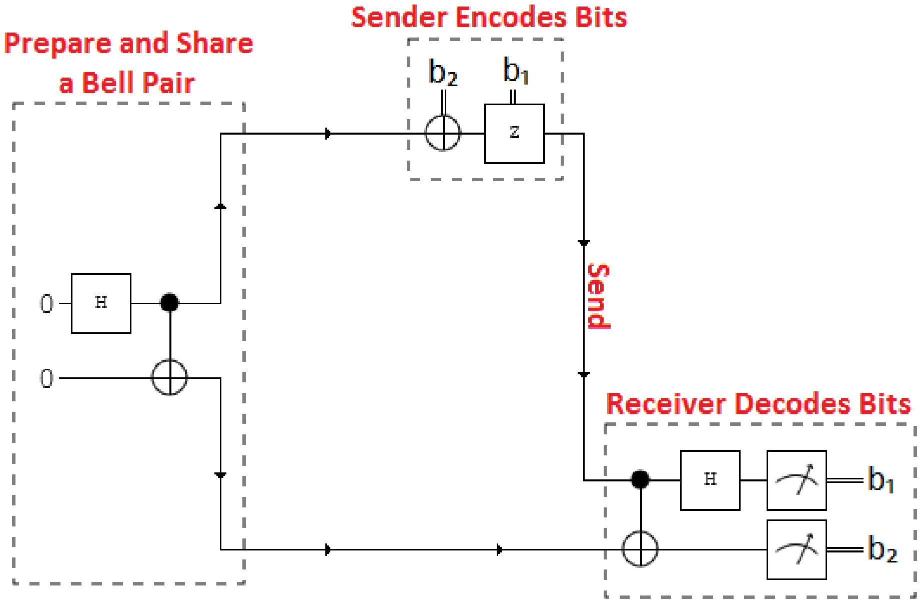


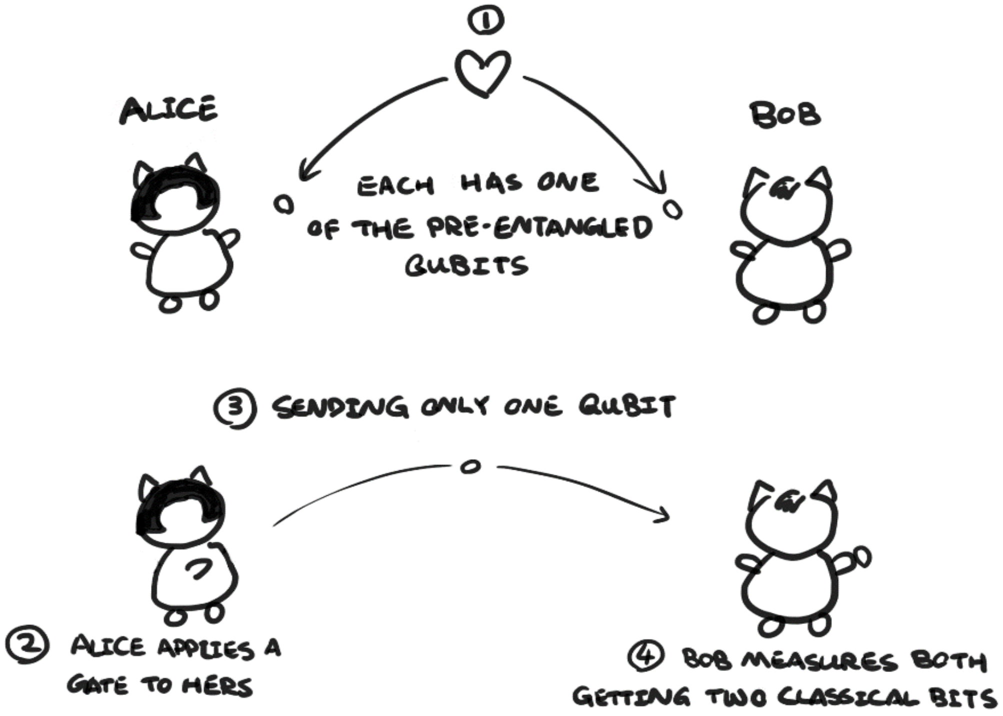
To send ‘01’, she applies an X gate

$$|\varphi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

Superdense coding

$$|\phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$





To send ‘01’, she applies an X gate

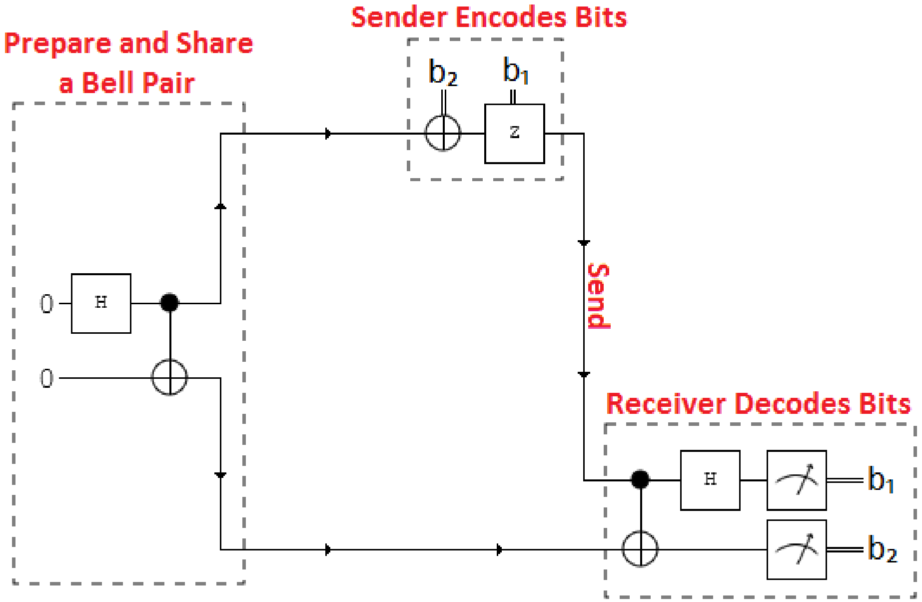
$$|\varphi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

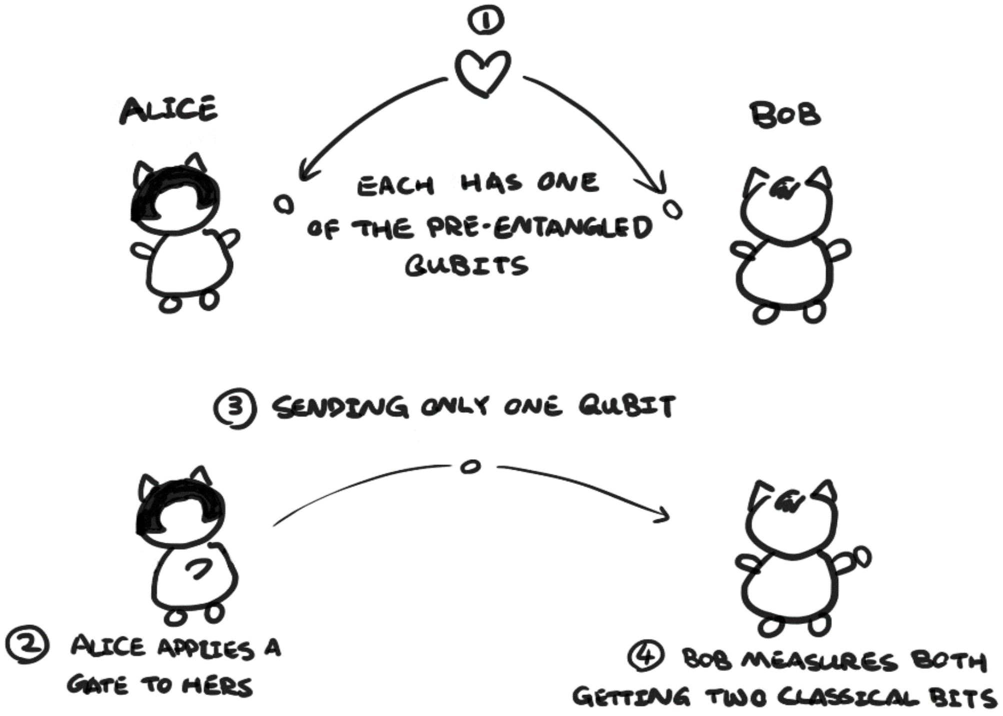
To send ‘10’, she applies a Z gate

$$|\phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

Superdense coding

$$|\phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$



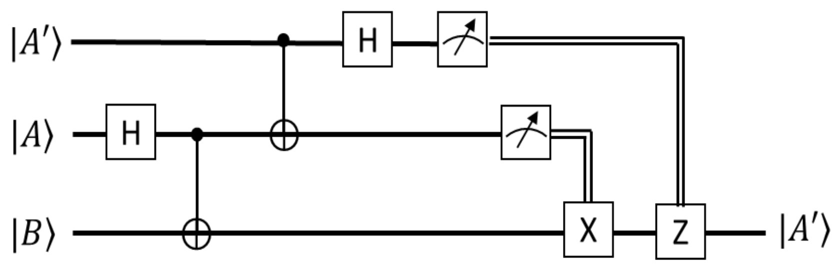


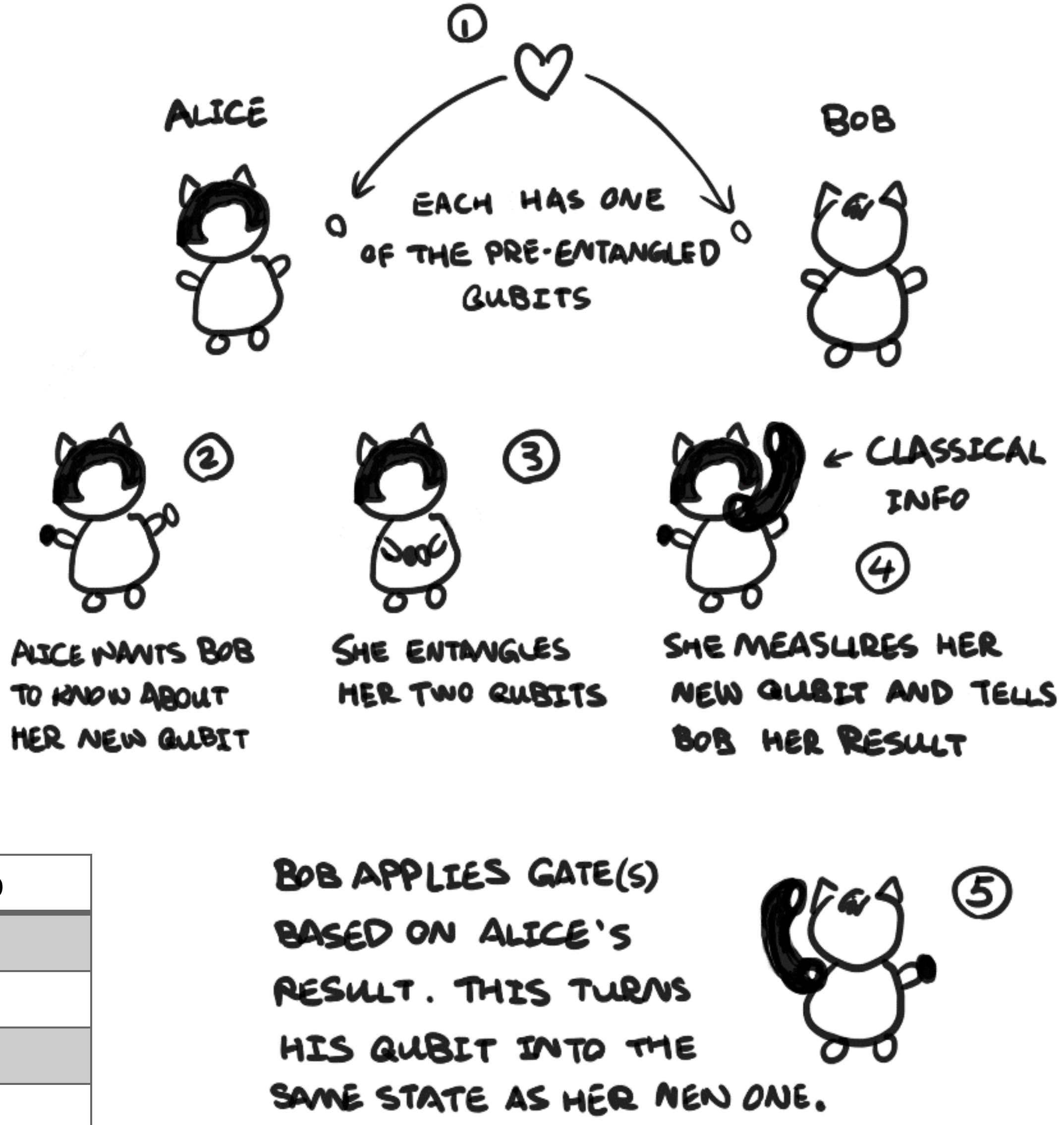
To send ‘01’, she applies an X gate $\quad |\varphi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$

To send ‘10’, she applies a Z gate $\quad |\phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$

For ‘11’, she uses an iY gate or a Z * X gate $\quad |\varphi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$

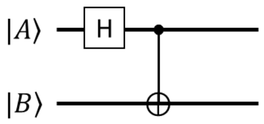
Teleportation





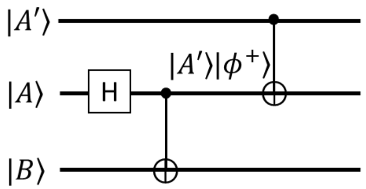
First two qubits	Third qubit	Alice tells Bob to
00	$[\alpha\|0\rangle + \beta\|1\rangle]$	do nothing
01	$[\alpha\|1\rangle + \beta\|0\rangle]$	apply X
10	$[\alpha\|0\rangle - \beta\|1\rangle]$	apply Z
11	$[\alpha\|1\rangle - \beta\|0\rangle]$	apply X and Z

Teleportation



$$|\phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

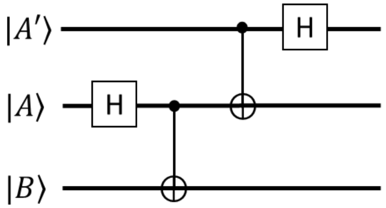
$$\boxed{\text{Let } |A'\rangle = \alpha|0\rangle + \beta|1\rangle}$$



$$|A'\rangle|\phi^+\rangle = (\alpha|0\rangle + \beta|1\rangle)\,\frac{|00\rangle+|11\rangle}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle).$$

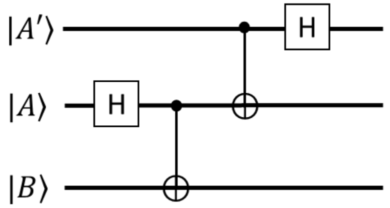
$$CNOT|A'\rangle|\phi^+\rangle = \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle)$$



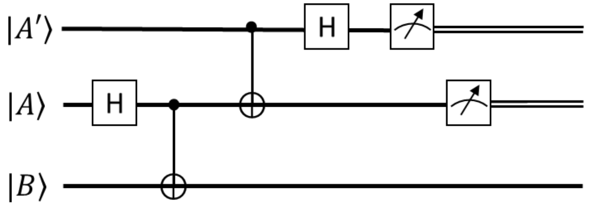
$$\frac{1}{2}[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)]$$

$$\frac{1}{\sqrt{2}}\left[\alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|00\rangle + \alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)|11\rangle + \beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)|10\rangle + \beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)|01\rangle\right]$$

Teleportation



$$\frac{1}{2}[|00\rangle(\alpha|0\rangle+\beta|1\rangle)+|01\rangle(\alpha|1\rangle+\beta|0\rangle)$$
$$+|10\rangle(\alpha|0\rangle-\beta|1\rangle)+|11\rangle(\alpha|1\rangle-\beta|0\rangle)]$$



If the first qubit is 0, the state after measurement becomes

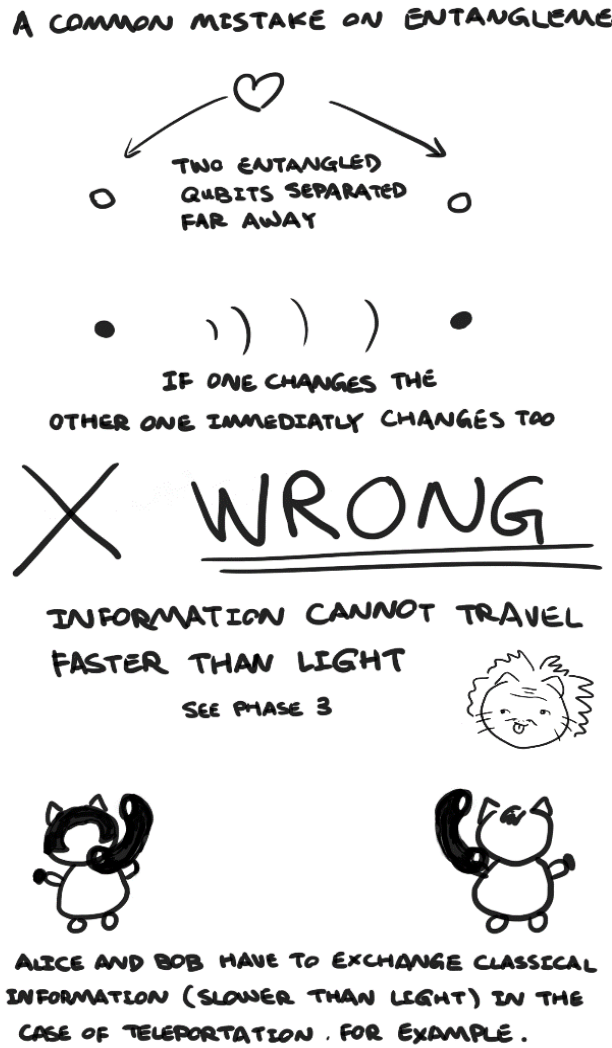
$$\frac{1}{2}[|00\rangle(\alpha|0\rangle+\beta|1\rangle)+|01\rangle(\alpha|1\rangle+\beta|0\rangle)] \, .$$

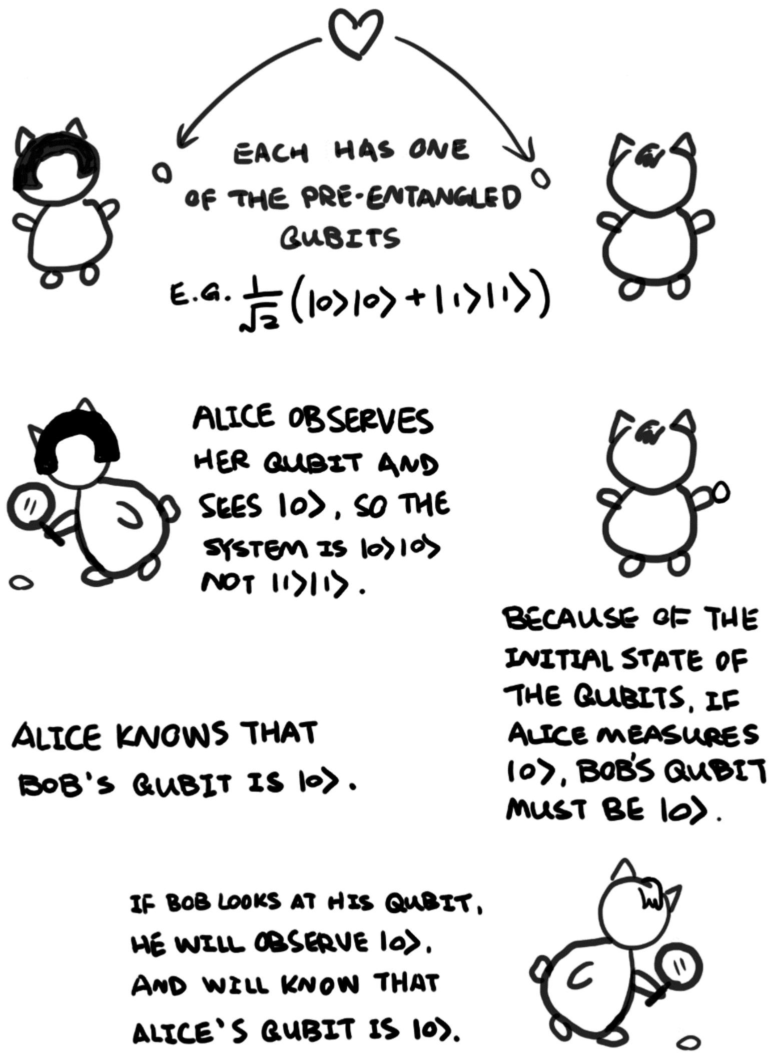
If then another measurement is done on the second qubit and it is 0, the state becomes

$$\frac{1}{2}[|00\rangle(\alpha|0\rangle+\beta|1\rangle)] \, .$$

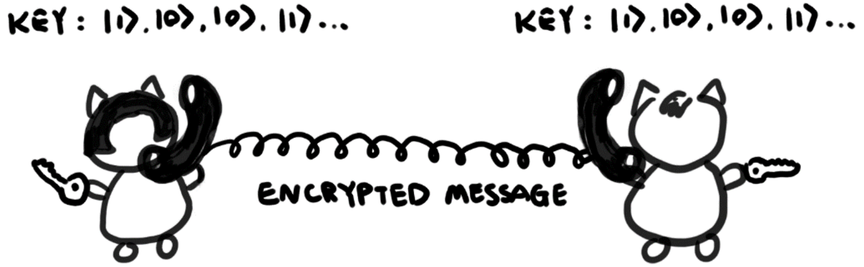
This also tells us that the third qubit is in state $[\alpha|0\rangle+\beta|1\rangle]$.

A common mistake





Encryption



They can’t communicate faster than light, but at least they can communicate securely.

Q# exercise: option 1

No installation, web-based Jupyter Notebooks

- The Quantum Katas project (tutorials and exercises for learning quantum computing) https://github.com/Microsoft/QuantumKatas

Q# exercise: option 2

Prerequisites

- Install VS Code and Quantum Development Kit extension according to instructions
- The Quantum Katas project (tutorials and exercises for learning quantum computing) https://github.com/Microsoft/QuantumKatas

Q# exercise: option 3

Prerequisites

- Please install Jupyter Notebooks and Q# following the instructions at https://docs.microsoft.com/quantum/install-guide#develop-with-jupyter-notebooks (any platform and any editor is fine)
- The Quantum Katas project (tutorials and exercises for learning quantum computing) https://github.com/Microsoft/QuantumKatas

Q# exercise: Single-qubit gates

1. Go to Basic Gates katas Task 1.1
2. Task 1.8

Q# exercise: Two-qubit gates

3. Task 2.1

Q# exercise: Superposition and Entanglement

1. Go to Superposition katas Task 4
2. Task 6
3. Try completing other tasks

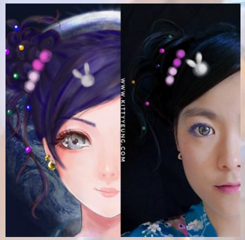
Q# exercise: Measurement

1. Go to Measurement katas Task 1.1 r
2. 1.3
3. Try completing other tasks

Q# exercise: Teleportation

1. Go to Teleportation katas Task 1.1-1.7
2. Try completing other tasks

Introduction to Quantum Computing



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